



Pearson International GCSE in Mathematics (Specification A) (4MA1)

Two-year Scheme of Work

For first teaching from September 2016

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Introduction

This scheme of work is based upon a five-term model over two years for both Foundation and Higher tier students. It can be used directly as a scheme of work for the International GCSE Mathematics (Specification A) (4MA1).

The scheme of work is broken up into two tiers, and then into units, so that there is greater flexibility for moving topics around to meet planning needs.

Each unit contains:

- Tier
- Contents, referenced back to the specification
- Prior knowledge
- Keywords.

Each sub-unit contains:

- Recommended teaching time, though of course this is adaptable according to individual teaching needs
- Objectives for students at the end of the sub-unit
- Possible success criteria for students at the end of the sub-unit
- Opportunities for reasoning/problem-solving
- Common misconceptions
- Notes for general mathematical teaching points.

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only.

International GCSE Mathematics

(Specification A)

Foundation Tier

Scheme of Work

| Unit number | | Title | Estimated teaching hours |
|--------------------------|----|--|--------------------------|
| Number | 1 | Integers and place value | 4 |
| | 2 | Decimals | 4 |
| | 3 | Special numbers and powers | 7 |
| | 4 | Fractions | 4 |
| | 5 | Percentages | 9 |
| | 6 | Ratio and proportion | 7 |
| | 7 | Arithmetic of fractions | 4 |
| | 8 | Set language, notation and Venn diagrams | 7 |
| | 9 | Indices and standard form | 5 |
| Algebra | 10 | Algebraic manipulation | 5 |
| | 11 | Expressions, formulae and rearranging formulae | 6 |
| | 12 | Linear equations and inequalities | 8 |
| | 13 | Sequences | 5 |
| | 14 | Real life graphs | 4 |
| | 15 | Linear graphs | 6 |
| | 16 | Quadratic equations and graphs | 5 |
| | 17 | Simultaneous equations | 4 |
| Space, shape and measure | 18 | Measures, bearings and scale drawings | 5 |
| | 19 | Symmetry, shapes, parallel lines and angle facts | 8 |
| | 20 | Interior and exterior angles of polygons | 5 |
| | 21 | Compound measures | 5 |
| | 22 | Perimeter, area and volume | 6 |
| | 23 | Circles and cylinders | 6 |
| | 24 | Transformations | 7 |
| | 25 | Pythagoras' theorem and Trigonometry | 12 |
| | 26 | Similarity and congruence in 2D | 5 |
| | 27 | Constructions | 4 |
| Handling Data | 28 | Graphical representation of data | 7 |
| | 29 | Statistical measures | 7 |
| | 30 | Probability | 9 |
| | | Total | 180 |

Number : Units 1 – 9

OBJECTIVES / SPECIFICATION REFERENCES

| Unit | Title | Specification Reference | Estimated teaching hours |
|------|----------------------------|--|--------------------------|
| 1 | Integers and place value | 1.1A understand and use integers (positive, negative and zero) | 4 |
| | | 1.1B understand place value | |
| | | 1.1C use directed numbers in practical situations | |
| | | 1.1D order integers | |
| | | 1.1E use the four rules of addition, subtraction, multiplication and division | |
| | | 1.1F use brackets and the hierarchy of operations | |
| | | 1.8A round integers to a given power of 10 | |
| 2 | Decimals | 1.3A use decimal notation | 4 |
| | | 1.3B understand place value | |
| | | 1.3C order decimals | |
| | | 1.3D convert a decimal to a fraction or percentage | |
| | | 1.3E recognise that a terminating decimal is a fraction | |
| | | 1.8B round to a given number of significant figures or decimal places | |
| | | 1.8C identify upper and lower bounds where values are given to a degree of accuracy | |
| | | 1.8D use estimation to evaluate approximations to numerical calculations | |
| | | 1.11A use a scientific electronic calculator to determine numerical results | |
| 3 | Special numbers and powers | 1.1G use the terms 'odd', 'even', 'prime numbers', 'factors' and 'multiples' | 7 |
| | | 1.1H identify prime factors, common factors and common multiples | |
| | | 1.4A identify square numbers and cube numbers | |
| | | 1.4B calculate squares, square roots, cubes and cube roots | |
| | | 1.4D express integers as product of powers of prime factors | |
| | | 1.4E find highest common factors (HCF) and lowest common multiples (LCM) | |
| 4 | Fractions | 1.2A understand and use equivalent fractions, simplifying a fraction by cancelling common factors | 4 |
| | | 1.2B understand and use mixed numbers and vulgar fractions | |
| | | 1.2C identify common denominators | |
| | | 1.2D order fractions and calculate a given fraction of a given quantity | |
| | | 1.2E express a given number as a fraction of another number | |
| | | 1.2G convert a fraction to a decimal or percentage | |

| Unit | Title | Specification Reference | Estimated teaching hours |
|------|--|--|--------------------------|
| 5 | Percentages | 1.6A understand that 'percentage' means 'number of parts per 100' | 9 |
| | | 1.6B express a given number as a percentage of another number | |
| | | 1.6C express a percentage as a fraction and as a decimal | |
| | | 1.6D understand the multiplicative nature of percentages as operators | |
| | | 1.6E solve simple percentage problems, including percentage increase and decrease | |
| | | 1.6F use reverse percentages | |
| | | 1.6G use compound interest and depreciation | |
| 6 | Ratio and proportion | 1.7A use ratio notation, including reduction to its simplest form and its various links to fraction notation | 7 |
| | | 1.7B divide a quantity in a given ratio or ratios | |
| | | 1.7C use the process of proportionality to evaluate unknown quantities | |
| | | 1.7D calculate an unknown quantity from quantities that vary in direct proportion | |
| | | 1.7E solve word problems about ratio and proportion | |
| | | 1.10A use and apply number in everyday personal, domestic or community life | |
| | | 1.10B carry out calculations using standard units of mass, length, area, volume and capacity | |
| | | 1.10C understand and carry out calculations using time, and carry out calculations using money, including converting between currencies | |
| 7 | Arithmetic of fractions | 1.2F use common denominators to add and subtract fractions and mixed numbers | 4 |
| | | 1.2H understand and use fractions as multiplicative inverses | |
| | | 1.2I multiply and divide fractions and mixed numbers | |
| 8 | Set language, notation and Venn diagrams | 1.5A understand the definition of a set | 7 |
| | | 1.5B use the set notation \cup , \cap and \in and \notin | |
| | | 1.5C understand the concept of the universal set and the empty set and the symbols for these sets | |
| | | 1.5D understand and use the complement of a set | |
| | | 1.5E use Venn diagrams to represent sets | |
| | | 6.3D find probabilities from a Venn diagram | |
| 9 | Indices and standard | 1.4C use index notation and index laws for multiplication and division of positive and negative integer powers including zero | 5 |

| | | | |
|--|------|--|--|
| | form | 1.9A calculate with and interpret numbers in the form $a \times 10^n$ where n is an integer and $1 \leq a < 10$ | |
|--|------|--|--|

Algebra : Units 10 – 17

OBJECTIVES / SPECIFICATION REFERENCES

| Unit | Title | Specification Reference | Estimated teaching hours |
|------|--|---|--------------------------|
| 10 | Algebraic manipulation | 2.1A understand that symbols may be used to represent numbers in equations or variables in expressions and formulae | 5 |
| | | 2.1B understand that algebraic expressions follow the generalised rules of arithmetic | |
| | | 2.1C use index notation for positive and negative integer powers (including zero) | |
| | | 2.1D use index laws in simple cases | |
| | | 2.2B collect like terms | |
| | | 2.2C multiply a single term over a bracket | |
| | | 2.2D take out common factors | |
| 11 | Expressions, formulae and rearranging formulae | 2.2A evaluate expressions by substituting numerical values for letters | 6 |
| | | 2.3A understand that a letter may represent an unknown number or a variable | |
| | | 2.3B use correct notational conventions for algebraic expressions and formulae | |
| | | 2.3C substitute positive and negative integers, decimals and fractions for words and letters in expressions and formulae | |
| | | 2.3D use formulae from mathematics and other real-life contexts expressed initially in words or diagrammatic form and convert to letters and symbols | |
| | | 2.3E derive a formula or expression | |
| | | 2.3F change the subject of a formula where the subject appears once | |
| | | | |
| 12 | Linear equations and inequalities | 2.4A solve linear equations, with integer or fractional coefficients, in one unknown in which the unknown appears on either side or both sides of the equation | 8 |
| | | 2.4B set up simple linear equations from given data | |
| | | 2.8A understand and use the symbols $>$, $<$, \leq and \geq | |
| | | 2.8B understand and use the convention for open and closed intervals on a number line | |

| | | | |
|-------------|--------------------------------|---|---------------------------------|
| | | 2.8C solve simple linear inequalities in one variable and represent the solution set on a number line | |
| | | | |
| Unit | Title | Specification Reference | Estimated teaching hours |
| 13 | Sequences | 3.1A generate terms of a sequence using term-to-term and position-to-term definitions of the sequence | 5 |
| | | 3.1B find subsequent terms of an integer sequence and the rule for generating it | |
| | | 3.1C use linear expressions to describe the n th term of arithmetic sequences | |
| 14 | Real life graphs | 3.3A interpret information presented in a range of linear and non-linear graphs | 4 |
| 15 | Linear graphs | 3.3B understand and use conventions for rectangular Cartesian coordinates | 6 |
| | | 3.3C plot points (x, y) in any of the four quadrants or locate points with given coordinates | |
| | | 3.3D determine the coordinates of points identified by geometrical information | |
| | | 3.3E determine the coordinates of the midpoint of a line segment, given the coordinates of the two end points | |
| | | 3.3F draw and interpret straight line conversion graphs | |
| | | 3.3G find the gradient of a straight line | |
| | | 3.3H recognise that equations of the form $y = mx + c$ are straight line graphs with gradient m and intercept on the y -axis at the point $(0, c)$ | |
| | | 3.3I recognise, generate points and plot graphs of linear functions | |
| | | 2.8D represent simple linear inequalities on rectangular Cartesian graphs | |
| | | 2.8E identify regions on rectangular Cartesian graphs defined by simple linear inequalities | |
| 16 | Quadratic equations and graphs | 2.2E expand the product of two simple linear expressions | 5 |
| | | 2.2F understand the concept of a quadratic expression and be able to factorise such expressions (limited to $x^2 + bx + c$) | |
| | | 2.7A solve quadratic equations by factorization (limited to $x^2 + bx + c = 0$) | |
| | | 3.3I recognise, generate points and plot graphs quadratic functions | |
| 17 | Simultaneous equations | 2.6A calculate the exact solution of two simultaneous equations in two unknowns | 4 |

Shape, space and measure : Units 18 - 27

OBJECTIVES / SPECIFICATION REFERENCES

| Unit | Title | Specification Reference | Estimated teaching hours |
|------|--|--|--------------------------|
| 18 | Measures, bearings and scale drawings | 4.4A interpret scales on a range of measuring instruments | 5 |
| | | 4.4B calculate time intervals in terms of the 24-hour and the 12-hour clock | |
| | | 4.4C make sensible estimates of a range of measures | |
| | | 4.4D understand angle measure including three-figure bearings | |
| | | 4.1A distinguish between acute, obtuse, reflex and right angles | |
| | | 4.4E measure an angle to the nearest degree | |
| | | 4.5A measure and draw lines to the nearest millimetre | |
| | | 4.5C solve problems using scale drawings | |
| | | 4.11B use and interpret maps and scale drawings | |
| | | 4.9A convert measurements within the metric system to include linear and area units | |
| | | 4.10A convert between units of volume within the metric system | |
| 19 | Symmetry, shapes, parallel lines and angle facts | 4.3A identify any lines of symmetry and the order of rotational symmetry of a given two-dimensional figure | 8 |
| | | 4.1B use angle properties of intersecting lines, parallel lines and angles on a straight line | |
| | | 4.1C understand the exterior angle of a triangle property and the angle sum of a triangle property | |
| | | 4.1D understand the terms 'isosceles', 'equilateral' and 'right-angled triangles' and the angle properties of these triangles | |
| | | 4.2B understand and use the term 'quadrilateral' and the angle sum property of quadrilaterals | |
| | | 4.2C understand and use the properties of the parallelogram, rectangle, square, rhombus, trapezium and kite | |
| | | 4.7A give informal reasons, where required, when arriving at numerical solutions to geometrical problems | |
| | | 4.10A recognise and give the names of solids | |
| | | 4.10B understand the terms 'face', 'edge' and 'vertex' in the context of 3-D solids | |

| Unit | Title | Specification Reference | Estimated teaching hours |
|------|----------------------------|--|--------------------------|
| 20 | Polygons | 4.2A recognise and give the names of polygons | 5 |
| | | 4.2D understand the term 'regular polygon' and calculate interior and exterior angles of regular polygons | |
| | | 4.2E understand and use the angle sum of polygons | |
| 21 | Compound measures | 4.4F understand and use the relationship between average speed, distance and time | 5 |
| | | 4.4G use compound measure such as speed, density and pressure | |
| 22 | Perimeter, area and volume | 4.9B find the perimeter of shapes made from triangles and rectangles | 6 |
| | | 4.9C find the area of simple shapes using the formulae for the areas of triangles and rectangles | |
| | | 4.9D find the area of parallelograms and trapezia | |
| | | 4.10C find the surface area of simple shapes using the area formulae for triangles and rectangles | |
| | | 4.10E find the volume of prisms, including cuboids and cylinders, using an appropriate formula | |
| 23 | Circles and cylinders | 4.6A recognise the terms 'centre', 'radius', 'chord', 'diameter', 'circumference', 'tangent', 'arc', 'sector' and 'segment' of a circle | 6 |
| | | 4.6B understand chord and tangent properties of circles | |
| | | 4.9E find circumferences and areas of circles using relevant formulae; find perimeters and areas of semicircles | |
| | | 4.10D find the surface area of a cylinder | |
| | | 4.10E find the volume of prisms, including cuboids and cylinders, using an appropriate formula | |

| Unit | Title | Specification Reference | Estimated teaching hours |
|------|--------------------------------------|--|--------------------------|
| 24 | Transformations | 5.2A understand that rotations are specified by a centre and an angle | 7 |
| | | 5.2B rotate a shape about a point through a given angle | |
| | | 5.2C recognise that an anti-clockwise rotation is a <i>positive</i> angle of rotation and a clockwise rotation is a <i>negative</i> angle of rotation | |
| | | 5.2D understand that reflections are specified by a mirror line | |
| | | 5.2E construct a mirror line given an object and reflect a shape given a mirror line | |
| | | 5.2F understand that translations are specified by a distance and direction | |
| | | 5.2G translate a shape | |
| | | 5.2H understand and use column vectors in translations | |
| | | 5.2I understand that rotations, reflections and translations preserve length and angle so that a transformed shape under any of these transformations remains congruent to the original shape | |
| | | 5.2J understand that enlargements are specified by a centre and a scale factor | |
| | | 5.2K understand that enlargements preserve angles and not lengths | |
| | | 5.2L enlarge a shape given the scale factor | |
| | | 5.2M identify and give complete descriptions of transformations | |
| 25 | Pythagoras' theorem and Trigonometry | 4.8A know, understand and use Pythagoras' Theorem in two dimensions | 12 |
| | | 4.8B know, understand and use sine, cosine and tangent of acute angles to determine lengths and angles of a right-angled triangle | |
| | | 4.8C apply trigonometrical methods to solve problems in two dimensions | |
| 26 | Similarity and congruence in 2D | 4.2F understand congruence as meaning the same shape and size | 5 |
| | | 4.2G understand that two or more polygons with the same shape and size are said to be congruent to each other | |
| | | 4.11A understand and use the geometrical properties that similar figures have corresponding lengths in the same ratio but corresponding angles remain unchanged | |
| 27 | Constructions and bearings | 4.5B construct triangles and other two-dimensional shapes using a combination of a ruler, a protractor and compasses | 4 |
| | | 4.5D use straight edge and compasses to: (i) construct the perpendicular bisector of a line segment (ii) construct the bisector of an angle | |

Handling Data : Units 28 – 30

OBJECTIVES / SPECIFICATION REFERENCES

| Unit | Title | Specification Reference | Estimated teaching hours |
|------|----------------------------------|---|--------------------------|
| 28 | Graphical representation of data | 6.1A use different methods of presenting data | 7 |
| | | 6.1B use appropriate methods of tabulation to enable the construction of statistical diagrams | |
| | | 6.1C interpret statistical diagrams | |
| 29 | Statistical measures | 6.2A understand the concept of average | 7 |
| | | 6.2B calculate the mean, median, mode and range for a discrete data set | |
| | | 6.2C calculate an estimate for the mean for grouped data | |
| | | 6.2D identify the modal class for grouped data | |
| 30 | Probability | 6.3A understand the language of probability | 9 |
| | | 6.3B understand and use the probability scale | |
| | | 6.3C understand and use estimates or measures of probability from theoretical models | |
| | | 6.3D find probabilities from a Venn diagram | |
| | | 6.3E understand the concepts of a sample space and an event, and how the probability of an event happening can be determined from the sample space | |
| | | 6.3F list all the outcomes for single events and for two successive events in a systematic way | |
| | | 6.3G estimate probabilities from previously collected data | |
| | | 6.3H calculate the probability of the complement of an event happening | |
| | | 6.3I use the addition rule of probability for mutually exclusive events | |
| | | 6.3J understand and use the term 'expected frequency' | |

1. Integers and place value**Teaching time**

3-5 hours

OBJECTIVES

| | |
|-------------|--|
| 1.1A | understand and use integers (positive, negative and zero) |
| 1.1B | understand place value |
| 1.1C | use directed numbers in practical situations |
| 1.1D | order integers |
| 1.1E | use the four rules of addition, subtraction, multiplication and division |
| 1.1F | use brackets and the hierarchy of operations |
| 1.8A | round integers to a given power of 10 |

POSSIBLE SUCCESS CRITERIA

Given 5 digits, what are the largest or smallest answers when subtracting a two-digit number from a three-digit number?

At noon the temperature is -4°C . At 3 pm the temperature has risen by 9°C . Find the temperature at 3pm.

Use inverse operations to justify answers, e.g. $9 \times 23 = 207$ so $207 \div 9 = 23$.

Check answers by rounding to nearest 10, 100, or 1000 as appropriate, e.g. $29 \times 31 \approx 30 \times 30$

Work out the value of $7 + 8 \div 2$; $(9 - 2) \times (5 + 1)$;

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Missing digits in calculations involving the four operations

Questions such as: Phil states $3.44 \times 10 = 34.4$ and Chris states $3.44 \times 10 = 34.40$. Who is correct?

Show me another number with 3, 4, 5, 6, 7 digits that includes a 6 with the same value as the "6" in the following number 36, 754

COMMON MISCONCEPTIONS

Stress the importance of knowing the multiplication tables to aid fluency.

Students may write statements such as $150 - 210 = 60$.

NOTES

Much of this unit will have been encountered by students in previous Key Stages, meaning that teaching time may focus on application or consolidation of prior learning.

Particular emphasis should be given to the importance of students presenting their work clearly.

Negative numbers in real life can be modelled by interpreting scales on thermometers using F and C.

Encourage the exploration of different calculation methods.

Students should be able to write numbers in words and from words as a real-life skill.

EXAMPLE QUESTIONS FROM SAMs : 2F q1, q3, q10a

2. Decimals**Teaching time**

3-5 hours

OBJECTIVES

| | |
|--------------|--|
| 1.3A | use decimal notation |
| 1.3B | understand place value |
| 1.3C | order decimals |
| 1.3D | convert a decimal to a fraction or percentage |
| 1.3E | recognise that a terminating decimal is a fraction |
| 1.8B | round to a given number of significant figures or decimal places |
| 1.8C | identify upper and lower bounds where values are given to a degree of accuracy |
| 1.8D | use estimation to evaluate approximations to numerical calculations |
| 1.11A | use a scientific electronic calculator to determine numerical results |

POSSIBLE SUCCESS CRITERIA

Order 0.06, 0.3, 0.63, 0.36, 0.603

Use mental methods for \times and \div , e.g. 5×0.6 , $1.8 \div 3$.

Solve a problem involving division by a decimal (up to 2 decimal places).

Given $2.6 \times 15.8 = 41.08$, what is 26×0.158 ? What is $4108 \div 26$?

Write 0.6 as a fraction, as a percentage

Round to 2 sig figs; 8756, 3.456, 0.05621, 567.9

Work out $\frac{2.3^2 + \sqrt{1.44}}{4.9 - 2.35}$, round your answer to 3 sig figs.

A length is 54 cm correct to the nearest cm. Write down the upper and lower bound of the length.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving shopping for multiple items, such as: Rob purchases a magazine costing £2.10, a newspaper costing 82p and two bars of chocolate. He pays with a £10 note and gets £5.40 change. Work out the cost of one bar of chocolate.

Explain why the answer to 6.58×2.4 cannot be 157.92

COMMON MISCONCEPTIONS

0.07 is bigger than 0.2

Significant figures and decimal place rounding are often confused.

Some students may think $35\ 877 = 36$ to two significant figures.

$\frac{45+67}{3}$ is often worked out incorrectly as $45 + 67 \div 3$ using a calculator.

NOTES

Practise estimating answers to calculations and use estimation as a method for checking answers.

Amounts of money should always be rounded to two decimal places (when appropriate).

EXAMPLE QUESTIONS FROM SAMs : 1F q6, q11

3. Special numbers and powers**Teaching time**

6-8 hours

OBJECTIVES

| | |
|-------------|---|
| 1.1G | use the terms 'odd', 'even', 'prime numbers', 'factors' and 'multiples' |
| 1.1H | identify prime factors, common factors and common multiples |
| 1.4A | identify square numbers and cube numbers |
| 1.4B | calculate squares, square roots, cubes and cube roots |
| 1.4D | express integers as product of powers of prime factors |
| 1.4E | find highest common factors (HCF) and lowest common multiples (LCM) |

POSSIBLE SUCCESS CRITERIA

What is the value of 2^3 ?

Work out the value of $3 + 2^4$

Recall prime numbers up to 100.

Find the HCF and LCM of 12 and 20

Write a number as a product of its prime factors.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide convincing counter-arguments to statements concerning properties of stated numbers, i.e. Sharon says 108 is a prime number. Is she correct?

Questions that require multiple layers of operations such as:

Pam writes down one multiple of 9 and two different factors of 40.

She then adds together her three numbers. Her answer is greater than 20 but less than 30.

Find three numbers that Pam could have written down.

COMMON MISCONCEPTIONS

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception. E.g. To work out $(-4)^2$, it is common to type -4^2 into a calculator and so get the incorrect answer of -16

Care is also needed when working with powers. E.g. 10^3 , is often interpreted as 10×3 .

1 is a prime number.

Particular emphasis should be made on the definition of 'product' as multiplication as many students get confused and think it relates to addition.

NOTES

Note that the students need to understand ,for example, $4\sqrt{2}$ as there will be occasions when their calculator displays an answer in surd form.

Use a number square to find primes (Eratosthenes sieve).

Using a calculator to check factors of large numbers can be useful.

Students need to be encouraged to learn squares from 2×2 to 15×15 and cubes of 2, 3, 4, 5 and 10 and corresponding square and cube roots.

EXAMPLE QUESTIONS FROM SAMs: 1F q1; 2F q16

4. Fractions**Teaching time**

3 - 5 hours

OBJECTIVES

| | |
|-------------|--|
| 1.2A | understand and use equivalent fractions, simplifying a fraction by cancelling common factors |
| 1.2B | understand and use mixed numbers and vulgar fractions |
| 1.2C | identify common denominators |
| 1.2D | order fractions and calculate a given fraction of a given quantity |
| 1.2E | express a given number as a fraction of another number |
| 1.2G | convert a fraction to a decimal or percentage |

POSSIBLE SUCCESS CRITERIA

Express a given number as a fraction of another, including where the fraction > 1 .

Simplify $\frac{120}{100}$.

Find $\frac{3}{5}$ of 15, $\frac{3}{4}$ of 20.

Find $\frac{1}{2}$ of 36 m, $\frac{1}{4}$ of £20.

Find the size of each category from a pie chart using fractions.

Write $\frac{2}{5}$ as (i) a decimal (ii) a percentage

Write $\frac{52}{6}$ as a mixed number in its simplest form.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions that involve rates of overtime pay including simple calculations involving fractional (>1 , e.g. 1.5) and hourly pay. These can be extended into calculating rates of pay given the final payment and number of hours worked.

Working out the number of people/things where the number of people/things in different categories is given as a fraction.

COMMON MISCONCEPTIONS

The larger the denominator the larger the fraction.

NOTES

When expressing a given number as a fraction of another, start with very simple numbers < 1 , and include some cancelling before fractions using numbers > 1 .

Regular revision of fractions is essential.

Demonstrate how to use the fraction button on the calculator.

Use real-life examples where possible.

EXAMPLE QUESTIONS FROM SAMs: 1F q2

5. Percentages**Teaching time**

8 - 10 hours

OBJECTIVES

| | |
|-------------|--|
| 1.6A | understand that 'percentage' means 'number of parts per 100' |
| 1.6B | express a given number as a percentage of another number |
| 1.6C | express a percentage as a fraction and as a decimal |
| 1.6D | understand the multiplicative nature of percentages as operators |
| 1.6E | solve simple percentage problems, including percentage increase and decrease |
| 1.6F | use reverse percentages |
| 1.6G | use compound interest and depreciation |

POSSIBLE SUCCESS CRITERIA

What is 10%, 15%, 17.5% of £30?

Write 64% as (i) a decimal (ii) as a fraction in its simplest form.

Jan's salary is £24 000. She gets a pay rise of 6%, work out her new salary.

Find the total interest is £4500 is invested for 3 years at 2.5% compound interest.

Normal prices are reduced by 15% in a sale. Find the normal price of an item with sale price £55.42

A car is bought for £2300 and sold for £4000. Find the percentage profit.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Sale prices offer an ideal opportunity for solving problems allowing students the opportunity to investigate the most effective way to work out the "sale" price.

Problems that involve consecutive reductions such as: Sale Prices are 10% off the previous day's price. If a jacket is £90 on Monday, what is the price on Wednesday?

COMMON MISCONCEPTIONS

It is not possible to have a percentage greater than 100%.

NOTES

Amounts of money should always be rounded to two decimal places.

Use real-life examples where possible.

Emphasise the importance of being able to convert between decimals and percentages and the use of decimal multipliers to make calculations easier.

EXAMPLE QUESTIONS FROM SAMs: 1F q2, q19, q23; 2F q20, q23

6. Ratio and proportion**Teaching time**

8 - 10 hours

OBJECTIVES

| | |
|--------------|---|
| 1.7A | use ratio notation, including reduction to its simplest form and its various links to fraction notation |
| 1.7B | divide a quantity in a given ratio or ratios |
| 1.7C | use the process of proportionality to evaluate unknown quantities |
| 1.7D | calculate an unknown quantity from quantities that vary in direct proportion |
| 1.7E | solve word problems about ratio and proportion |
| 1.10A | use and apply number in everyday personal, domestic or community life |
| 1.10B | carry out calculations using standard units of mass, length, area, volume and capacity |
| 1.10C | understand and carry out calculations using time, and carry out calculations using money, including converting between currencies |

POSSIBLE SUCCESS CRITERIA

Write a ratio to describe a situation such as 1 blue for every 2 red, or 3 adults for every 10 children.

Share \$98 in the ratio 2 : 3 : 5

If £1 = \$1.42, how many \$ do you get for £50; how many £ do you get for \$67 ?

Scale up recipes and decide if there is enough of each ingredient.

Given two sets of data in a table, are they in direct proportion?

A film starts at 11:50 and ends at 13:35, how long did it last?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Anna, Bob and Clive share some money in the ratio 1 : 2 : 4. Clive gets £36 more than Anna. How much did Bob get?

Problems in context, such as scaling a recipe, or diluting lemonade or chemical solutions, will show how proportional reasoning is used in real-life contexts.

COMMON MISCONCEPTIONS

Using a ratio to find one quantity when the other is known often results in students 'sharing' the known amount.

NOTES

Emphasise the importance of reading the question carefully.

Include ratios with decimals 0.2 : 1.

Find out/prove whether two variables are in direct proportion by plotting the graph and using it as a model to read off other values.

EXAMPLE QUESTIONS FROM SAMs: 1F q15, q17; 2F q10, q15

7. Arithmetic of fractions**Teaching time**

3 - 5 hours

OBJECTIVES

| | |
|-------------|---|
| 1.2F | use common denominators to add and subtract fractions and mixed numbers |
| 1.2H | understand and use fractions as multiplicative inverses |
| 1.2I | multiply and divide fractions and mixed numbers |

POSSIBLE SUCCESS CRITERIA

$$\frac{3}{5} \times 15, 20 \times \frac{3}{4}.$$

$$\frac{1}{2} \text{ of } 36 \text{ m, } \frac{1}{4} \text{ of } £20.$$

$$\text{Calculate: } \frac{1}{2} \times \frac{6}{7}, \frac{3}{5} \div 3.$$

$$\text{Work out } 2\frac{2}{3} + 1\frac{4}{5}; 2\frac{2}{3} - 1\frac{4}{5}; 2\frac{2}{3} \times 1\frac{4}{5}; 2\frac{2}{3} \div 1\frac{4}{5}$$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions that involve rates of overtime pay including simple calculations involving fractional (>1 , e.g. 1.5) and hourly pay. These can be extended into calculating rates of pay given the final payment and number of hours worked.

Working out the number of people/things where the number of people/things in different categories is given as a fraction, decimal or percentage.

COMMON MISCONCEPTIONS

The larger the denominator the larger the fraction.

You add fractions by adding the numerators and then the denominators.

NOTES

When adding and subtracting fractions, start with same denominator, then where one denominator is a multiple of the other (answers ≤ 1), and finally where both denominators have to be changed (answers ≤ 1).

Regular revision of fractions is essential.

Demonstrate how to use the fraction button on the calculator.

Use real-life examples where possible.

EXAMPLE QUESTIONS FROM SAMs: 2F q25

8. Set language, notation and Venn diagrams**Teaching time**

6 - 8 hours

OBJECTIVES

| | |
|-------------|--|
| 1.5A | understand the definition of a set |
| 1.5B | use the set notation \cup , \cap and \in and \notin |
| 1.5C | understand the concept of the universal set and the empty set and the symbols for these sets |
| 1.5D | understand and use the complement of a set |
| 1.5E | use Venn diagrams to represent sets |
| 6.3D | find probabilities from a Venn diagram |

POSSIBLE SUCCESS CRITERIA

Universal set is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$; Write down $A \cap B$, $A \cup B$

$C = \{1, 3, 5\}$; write down C'

Is $4 \in C$, is $4 \in A$

Draw a Venn diagram to show the universal set, A and B ;

If a number is picked at random, find $P(A \cap B)$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given Universal set is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{5, 7, 9\}$ and $B = \{1, 3, 5, 7\}$

Write down a possible set C so that $A \cap C = \{7\}$ and C has 4 members

COMMON MISCONCEPTIONS

$A = \{5, 7, 9\}$ and $B = \{1, 3, 5, 7\}$ then $A \cup B = \{1, 3, 5, 5, 7, 7, 9\}$

NOTES

When drawing a Venn diagram it is a good idea to put members in the intersection first.

EXAMPLE QUESTIONS FROM SAMs: -

9. Indices and standard form**Teaching time**

4-6 hours

OBJECTIVES

| | |
|-------------|--|
| 1.4C | use index notation and index laws for multiplication and division of positive and negative integer powers including zero |
| 1.9A | calculate with and interpret numbers in the form $a \times 10^n$ where n is an integer and $1 \leq a < 10$ |

POSSIBLE SUCCESS CRITERIA

Write 51 080 in standard form.

Write 3.74×10^{-6} as an ordinary number.

What is 9^0 ?

Simplify $6^9 \times 6^{13}$; $4^{12} \div 4^2$;

Evaluate $(2^{-3} \times 2^5) \div 2^4$.

Write, as a single power of 7, $7^{13} \times 7^5$

Work out $(1.2 \times 10^4) \times (3 \times 10^{-9})$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Link with other areas of mathematics, such as compound measures, by using speed of light in standard form.

COMMON MISCONCEPTIONS

Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form.

NOTES

Standard form is used in science and there are lots of cross curricular opportunities.

Students need to be provided with plenty of practice in using standard form with calculators.

EXAMPLE QUESTIONS FROM SAMs: 1F q24

10. Algebraic manipulation**Teaching time**

4-6 hours

OBJECTIVES

| | |
|-------------|--|
| 2.1A | understand that symbols may be used to represent numbers in equations or variables in expressions and formulae |
| 2.1B | understand that algebraic expressions follow the generalised rules of arithmetic |
| 2.1C | use index notation for positive and negative integer powers (including zero) |
| 2.1D | use index laws in simple cases |
| 2.2B | collect like terms |
| 2.2C | multiply a single term over a bracket |
| 2.2D | take out common factors |

POSSIBLE SUCCESS CRITERIASimplify $4p - 2q + 3p + 5q$ Simplify $5(a + 2b) - 3(3a - b)$ Expand $5(2x + 3)$; $x(x + 2)$ Factorise $18a + 27$; $a^2 + 3a$; $12m^3 + 9m^2$ Simplify $z^4 \times z^3$, $y^3 \div y^2$, $(a^7)^2 p^0$ Simplify $x^{-4} \times x^2$, $w^2 \div w^{-1}$.**OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Forming expressions and equations using area and perimeter of 2D shapes.

COMMON MISCONCEPTIONS

Any poor number skills involving negatives and times tables will become evident.

A common misconception is $3(x + 4) = 3x + 4$.The convention of not writing a coefficient with a single value, i.e. x instead of $1x$, may cause confusion.**NOTES**Emphasise correct use of symbolic notation, i.e. $3 \times y = 3y$ and not $y3$ and $a \times b = ab$.

Use lots of concrete examples when writing expressions, e.g. 'B' boys + 'G' girls.

Plenty of practice should be given and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed.

EXAMPLE QUESTIONS FROM SAMs: 1F q7a, q14a, q21a; 2F q9abf, 19abc

11. Expressions, formulae and rearranging equations**Teaching time**

5-7 hours

OBJECTIVES

| | |
|-------------|---|
| 2.2A | evaluate expressions by substituting numerical values for letters |
| 2.3A | understand that a letter may represent an unknown number or a variable |
| 2.3B | use correct notational conventions for algebraic expressions and formulae |
| 2.3C | substitute positive and negative integers, decimals and fractions for words and letters in expressions and formulae |
| 2.3D | use formulae from mathematics and other real-life contexts expressed initially in words or diagrammatic form and convert to letters and symbols |
| 2.3E | derive a formula or expression |
| 2.3F | change the subject of a formula where the subject appears once |

POSSIBLE SUCCESS CRITERIA

Evaluate the expressions for different values of x : $3x^2 + 4$ or $2x^3$.

There are 6 eggs in a small box and 12 eggs in a large box. Gary buys s small boxes and g large boxes. Write down an expression for the total number of eggs Gary buys.

Make t the subject of $v = u + at$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming and solving equations involving algebra and other areas of mathematics such as area and perimeter.

COMMON MISCONCEPTIONS

If $a = 2$ sometimes students interpret $3a$ as 32.

Making mistakes with negatives, including the squaring of negative numbers.

NOTES

Provide students with lots of practice.

This topic lends itself to regular reinforcement through starters in lessons.

Use formulae from mathematics and other subjects, expressed initially in words and then using letters and symbols.

EXAMPLE QUESTIONS FROM SAMs: 1F q8; 2F q9de

12. Equations and inequalities**Teaching time**

7-9 hours

OBJECTIVES

| | |
|-------------|---|
| 2.4A | solve linear equations, with integer or fractional coefficients, in one unknown in which the unknown appears on either side or both sides of the equation |
| 2.4B | set up simple linear equations from given data |
| 2.8A | understand and use the symbols $>$, $<$, \square and \square |
| 2.8B | understand and use the convention for open and closed intervals on a number line |
| 2.8C | solve simple linear inequalities in one variable and represent the solution set on a number line |

POSSIBLE SUCCESS CRITERIA

Solve: $x + 5 = 12$, $x - 6 = 3$, $\frac{x}{2} = 5$, $2x - 5 = 19$, $2x + 5 = 8x - 7$

Given expressions for the angles on a line or in a triangle in terms of a , find the value of a .

Given expressions for the sides of a rectangle and the perimeter, form and solve an equation to find missing values.

Solve $-3 < 2x + 1$ and show the solution set on a number line.

State the whole numbers that satisfy a given inequality.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that:

- could be solved by forming equations such as: Pat and Paul have a combined salary of £800 per week. Pat earns £200 per week more than Paul. How much does Paul earn?
- involve the application of a formula with conflicting results such as: Pat and Paul are using the formula $y = 8n + 4$. When $n = 2$, Pat states that $y = 86$ and Paul states $y = 20$. Who is correct?

COMMON MISCONCEPTIONS

Rules of adding and subtracting negatives.

Inverse operations can be misapplied.

When solving inequalities, students often state their final answer as a number quantity and either exclude the inequality or change it to $=$.

NOTES

Emphasise good use of notation.

Students need to realise that not all linear equations can be solved by observation or trial and improvement, and hence the use of a formal method is important.

Students can leave their answer in fraction form where appropriate.

Emphasise the importance of leaving their answer as an inequality (and not change to $=$).

EXAMPLE QUESTIONS FROM SAMs: 1F q7b; 2F q9c, q19d

13. Sequences**Teaching time**

4-6 hours

OBJECTIVES

| | |
|-------------|--|
| 3.1A | generate terms of a sequence using term-to-term and position-to-term definitions of the sequence |
| 3.1B | find subsequent terms of an integer sequence and the rule for generating it |
| 3.1C | use linear expressions to describe the n th term of arithmetic sequences |

POSSIBLE SUCCESS CRITERIA

Given a sequence, 'Which is the 1st term greater than 50?'

What is the amount of money after x months saving the same amount or the height of tree that grows 6 m per year?

What are the next terms in the following sequences?

1, 3, 9, ... 100, 50, 25, ... 2, 4, 8, 16, ...

Write down an expression for the n th term of the arithmetic sequence 2, 5, 8, 11, ...

Is 67 a term in the sequence 4, 7, 10, 13, ...?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluating statements about whether or not specific numbers or patterns are in a sequence and justifying the reasons.

COMMON MISCONCEPTIONS

The n th term of the sequence 1, 4, 7, 10 ... is $n + 3$ (rather than $3n - 2$)

NOTES

Emphasise use of $3n$ meaning $3 \times n$.

Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the n th term.

Students are not expected to find the n th term of a quadratic sequence.

EXAMPLE QUESTIONS FROM SAMs: 1F q4; 2F q17

14. Real-life graphs**Teaching time**

3-5 hours

OBJECTIVES

| | |
|-------------|--|
| 3.3A | interpret information presented in a range of linear and non-linear graphs |
|-------------|--|

POSSIBLE SUCCESS CRITERIA

Interpret a description of a journey into a distance–time or speed–time graph.
 Read information from a distance-time or speed-time graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to decide what the scales on any axis should be to be able to draw a correct graph.
 Conversion graphs can be used to provide opportunities for students to justify which distance is further, or whether or not certain items can be purchase in different currencies.

COMMON MISCONCEPTIONS

With distance–time graphs, students struggle to understand that the perpendicular distance from the x -axis represents distance.

NOTES

Clear presentation of axes is important.
 Ensure that you include questions that include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.
 Careful annotation should be encouraged: it is good practice to get the students to check that they understand the increments on the axes.
 Use standard units of measurement to draw conversion graphs.
 Use various measures in distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours.

EXAMPLE QUESTIONS FROM SAMs: 2F q5

15. Straight-line graphs**Teaching time**

5–7 hours

OBJECTIVES

| | |
|-------------|---|
| 3.3B | understand and use conventions for rectangular Cartesian coordinates |
| 3.3C | plot points (x, y) in any of the four quadrants or locate points with given coordinates |
| 3.3D | determine the coordinates of points identified by geometrical information |
| 3.3E | determine the coordinates of the midpoint of a line segment, given the coordinates of the two end points |
| 3.3F | draw and interpret straight line conversion graphs |
| 3.3G | find the gradient of a straight line |
| 3.3H | recognise that equations of the form $y = mx + c$ are straight line graphs with gradient m and intercept on the y -axis at the point $(0, c)$ |
| 3.3I | recognise, generate points and plot graphs of linear functions |
| 2.8D | represent simple linear inequalities on rectangular Cartesian graphs |
| 2.8E | identify regions on rectangular Cartesian graphs defined by simple linear inequalities |

POSSIBLE SUCCESS CRITERIA

Be able to plot points (or write down coordinates) in all quadrants.

Use a conversion graph.

Plot and draw the graph for $y = 2x - 4$

Which of these lines are parallel: $y = 2x + 3$, $y = 5x + 3$, $y = 2x - 9$, $2y = 4x - 8$

Show region satisfied by $x \geq -1$; $y < 5$; $x + y < 3$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given three vertices of a parallelogram, find coordinates of the fourth vertex.

Students should be able to decide what the scales on any axis should be in order to draw a correct graph.

Use a conversion graph to convert quantities that cannot be found on the axes. Eg. scale goes from 1 kg to 10 kg; convert 150 kg into pounds.

COMMON MISCONCEPTIONS

When not given a table of values, students rarely see the relationship between the coordinate axes.

NOTES

Emphasise the importance of drawing a table of values when not given one.

Values for a table should be taken from the x -axis.

EXAMPLE QUESTIONS FROM SAMs: 1F q12; 2F q14

16. Quadratic equations and graphs**Teaching time**

4–6 hours

OBJECTIVES

| | |
|-------------|---|
| 2.2E | expand the product of two simple linear expressions |
| 2.2F | understand the concept of a quadratic expression and be able to factorise such expressions (limited to $x^2 + bx + c$) |
| 2.7A | solve quadratic equations by factorization (limited to $x^2 + bx + c = 0$) |
| 3.3I | recognise, generate points and plot graphs quadratic functions |

POSSIBLE SUCCESS CRITERIASolve $3x^2 + 4 = 100$ Expand $(x + 2)(x + 6)$ Factorise $x^2 + 7x + 10$ Solve $x^2 + 7x + 10 = 0$ Solve $(x - 3)(x + 4) = 0$

Recognise a linear graph from its shape.

Recognise a quadratic graph from its shape.

Draw the graph of $y = x^2 + 3x - 4$ **OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Visual proof of the difference of two squares.

Given the length and width of a rectangle as expressions in x and the area of the rectangle, form a quadratic equation.**COMMON MISCONCEPTIONS** x terms can sometimes be 'collected' with x^2 .

Squaring negative numbers can be a problem.

NOTESEmphasise the fact that x^2 and x are different 'types' of term – illustrate this with numbers.

The graphs should be drawn freehand and in pencil, joining points using a smooth curve.

Encourage efficient use of the calculator.

EXAMPLE QUESTIONS FROM SAMs: 1F q21b

17. Simultaneous Equations**Teaching time**

3–5 hours

OBJECTIVES

| | |
|-------------|--|
| 2.6A | calculate the exact solution of two simultaneous equations in two unknowns |
|-------------|--|

POSSIBLE SUCCESS CRITERIA

Solve two simultaneous equations in two variables (linear/linear) algebraically

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Simple simultaneous equations can be formed and solved from real life scenarios, such as 2 adult and 2 child tickets cost £18, and 1 adult and 3 child tickets costs £17. What is the cost of 1 adult ticket?

COMMON MISCONCEPTIONS

The values of variables must be integer.

NOTES

Emphasise the need for good algebraic notation.
Clear algebraic working must be shown.

EXAMPLE QUESTIONS FROM SAMs: 2F q24

18. Measures, bearings and scale drawings**Teaching time**

4-6 hours

OBJECTIVES

| | |
|--------------|--|
| 4.4A | interpret scales on a range of measuring instruments |
| 4.4B | calculate time intervals in terms of the 24-hour and the 12-hour clock |
| 4.4C | make sensible estimates of a range of measures |
| 4.4D | understand angle measure including three-figure bearings |
| 4.1A | distinguish between acute, obtuse, reflex and right angles |
| 4.4E | measure an angle to the nearest degree |
| 4.5A | measure and draw lines to the nearest millimetre |
| 4.5C | solve problems using scale drawings |
| 4.11B | use and interpret maps and scale drawings |
| 4.9A | convert measurements within the metric system to include linear and area units |
| 4.10A | convert between units of volume within the metric system |

POSSIBLE SUCCESS CRITERIA

Film starts at 13:50 and ends at 15:10; how long was the film?

Measure an angle to the nearest degree.

Measure a line; give your answer in mm.

Change 5.6kg to grams; 56 mm to cm

Change 3 m² to cm²; 5 cm³ to mm³

Use AB notation for describing lengths and $\angle ABC$ notation for describing angles.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Work out a speed, having first had to work out a time.

Work out cost of 400 g of cheese given price of 1 kg of cheese.

COMMON MISCONCEPTIONS

Using the wrong scale on a protractor. E.g. measuring an angle of 50° as 130°

Using the wrong conversion factor from e.g. mm to cm or m to km.

2 hours 30 minutes is written as 2.3 in decimal form.

NOTES

Emphasise that diagrams in examinations are seldom drawn accurately.

Make sure drawings are neat, labelled and accurate.

Give students lots of practice.

Angles should be accurate to within 2°.

Use tracing paper to assist with symmetry questions.

Ask students to find their own examples of symmetry in real life.

EXAMPLE QUESTIONS FROM SAMs: 2F q13

19. Symmetry, shapes, parallel lines and angle facts**Teaching time**

7-9 hours

OBJECTIVES

| | |
|--------------|--|
| 4.3A | identify any lines of symmetry and the order of rotational symmetry of a given two-dimensional figure |
| 4.1B | use angle properties of intersecting lines, parallel lines and angles on a straight line |
| 4.1C | understand the exterior angle of a triangle property and the angle sum of a triangle property |
| 4.1D | understand the terms 'isosceles', 'equilateral' and 'right-angled triangles' and the angle properties of these triangles |
| 4.2B | understand and use the term 'quadrilateral' and the angle sum property of quadrilaterals |
| 4.2C | understand and use the properties of the parallelogram, rectangle, square, rhombus, trapezium and kite |
| 4.7A | give informal reasons, where required, when arriving at numerical solutions to geometrical problems |
| 4.10A | recognise and give the names of solids |
| 4.10B | understand the terms 'face', 'edge' and 'vertex' in the context of 3-D solids |

POSSIBLE SUCCESS CRITERIA

Name all quadrilaterals that have a specific property.

Use geometric reasoning to answer problems giving detailed reasons.

Find the size of missing angles at a point or at a point on a straight line.

Identify lines of symmetry, order of rotational symmetry of a given shape.

Give the number of faces, edges and vertices of a cuboid.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step "angle chasing" style problems that involve justifying how students have found a specific angle.

Geometrical problems involving algebra whereby equations can be formed and solved allow students the opportunity to make and use connections with different parts of mathematics.

What is the same, and what is different between families of polygons?

COMMON MISCONCEPTIONS

Some students will think that all trapezia are isosceles, or a square is only square if 'horizontal', or a 'non-horizontal' square is called a diamond.

Incorrectly identifying the 'base angles' (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.

Misunderstanding angle notation such as angle ABC

NOTES

Emphasise that diagrams in examinations are seldom drawn accurately.

Write any found angles on the diagram in a question and/or identify clearly in working.

Emphasise the need to give geometric reasons when required.

EXAMPLE QUESTIONS FROM SAMs: 1F q5a, q9

20. Polygons**Teaching time**

4-6 hours

OBJECTIVES

| | |
|-------------|--|
| 4.2A | recognise and give the names of polygons |
| 4.2D | understand the term 'regular polygon' and calculate interior and exterior angles of regular polygons |
| 4.2E | understand and use the angle sum of polygons |

POSSIBLE SUCCESS CRITERIA

Deduce and use the angle sum in any polygon.

Derive the angle properties of regular polygons.

Given the size of its exterior angle, how many sides does the polygon have?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems whereby students have to justify the number of sides that a regular polygon has given an interior or exterior angle.

COMMON MISCONCEPTIONS

Pupils may believe, incorrectly, that all polygons are regular.

NOTES

Study Escher drawings.

Use examples of tiling patterns with simple shapes to help students investigate if shapes 'fit together'.

EXAMPLE QUESTIONS FROM SAMs: 2F q26

21. Compound Measure**Teaching time**

4-6 hours

OBJECTIVES

| | |
|-------------|--|
| 4.4F | understand and use the relationship between average speed, distance and time |
| 4.4G | use compound measure such as speed, density and pressure |

POSSIBLE SUCCESS CRITERIA

Find the speed given distance and time.

Find the distance (in km) given the speed (in km/h) and the time (in minutes).

Recall and use the formula for density.

Give the formula for pressure, use it to find one of the variables.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the mass of an object, having first to find its volume.

Work out the average speed of a journey.

COMMON MISCONCEPTIONS

Using inconsistent units when solving problems.

Converting time into a decimal incorrectly. E.g. writing 1 hour 15 minutes as 1.15 hours.

NOTES

Practice converting time into decimals.

Ensure that conversions between metric units are known.

EXAMPLE QUESTIONS FROM SAMs: 1F q16; 2F q18

22. Perimeter, area and volume**Teaching time**

5-7 hours

OBJECTIVES

| | |
|--------------|---|
| 4.9B | find the perimeter of shapes made from triangles and rectangles |
| 4.9C | find the area of simple shapes using the formulae for the areas of triangles and rectangles |
| 4.9D | find the area of parallelograms and trapezia |
| 4.10C | find the surface area of simple shapes using the area formulae for triangles and rectangles |
| 4.10E | find the volume of prisms, including cuboids and cylinders, using an appropriate formula |

POSSIBLE SUCCESS CRITERIA

Find the area/perimeter of a given shape, stating the correct units.

Justify whether a certain number of small boxes fit inside a larger box.

Calculate the volume of a triangular prism with correct units.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given two 2D that shapes have equal areas, work out all the dimensions of the sides of the shapes.

Problems involving straight-forward and compound shapes in a real-life context should be explored to reinforce the concept of area. For example, the plan of a garden linked to the purchase of grass seed.

COMMON MISCONCEPTIONS

Shapes involving missing lengths of sides often result in incorrect answers.

Students often confuse perimeter and area.

Volume often gets confused with surface area.

NOTES

Use questions that involve different metric measures that need converting.

Measurement is essentially a practical activity: use a range of everyday shapes to bring reality to lessons.

Ensure that students are clear about the difference between perimeter and area.

Practical examples help to clarify the concepts, i.e. floor tiles, skirting board, etc.

Discuss the correct use of units.

Drawings should be done in pencil.

Consider 'how many small boxes fit in a larger box'-type questions.

Practical examples should be used to enable students to understand the difference between perimeter, area and volume.

EXAMPLE QUESTIONS FROM SAMs: 1F q5bc, q10, q25; 2F q12, q18

23. Circles and cylinders**Teaching time**

5-7 hours

OBJECTIVES

| | |
|--------------|--|
| 4.6A | recognise the terms 'centre', 'radius', 'chord', 'diameter', 'circumference', 'tangent', 'arc', 'sector' and 'segment' of a circle |
| 4.6B | understand chord and tangent properties of circles |
| 4.9E | find circumferences and areas of circles using relevant formulae; find perimeters and areas of semicircles |
| 4.10D | find the surface area of a cylinder |
| 4.10E | find the volume of prisms, including cuboids and cylinders, using an appropriate formula |

POSSIBLE SUCCESS CRITERIA

Recall terms related to a circle.

Understand that answers in terms of pi are more accurate.

Find the volume of a cylinder given the height and diameter.

Find the area and circumference of a circle given the radius or diameter.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Calculate the radius/diameter given the area/circumference type questions could be explored, including questions that require evaluation of statements, such as Andy states "Diameter = $2 \times$ Radius" and Bob states "Radius = $2 \times$ Diameter". Who is correct?

Problems involving straight-forward and compound shapes in a real-life context should be explored to reinforce the concept of area. For example, the floor plan of a room linked to the amount of flooring needed.

Problems using number of revolutions of a wheel.

COMMON MISCONCEPTIONS

Diameter and radius are often confused and recollection which formula to use for area and circumference of circles is often poor.

Volume often gets confused with surface area.

NOTES

Emphasise the need to learn the circle formula: 'Cherry Pie's Delicious' and 'Apple Pies are too' are good ways to remember them.

Ensure that students know it is more accurate to leave answers in terms of π but only when asked to do so.

EXAMPLE QUESTIONS FROM SAMs: 1F q25

24. Transformations**Teaching time**

6-8 hours

OBJECTIVES

| | |
|-------------|--|
| 5.2A | understand that rotations are specified by a centre and an angle |
| 5.2B | rotate a shape about a point through a given angle |
| 5.2C | recognise that an anti-clockwise rotation is a <i>positive</i> angle of rotation and a clockwise rotation is a <i>negative</i> angle of rotation |
| 5.2D | understand that reflections are specified by a mirror line |
| 5.2E | construct a mirror line given an object and reflect a shape given a mirror line |
| 5.2F | understand that translations are specified by a distance and direction |
| 5.2G | translate a shape |
| 5.2H | understand and use column vectors in translations |
| 5.2I | understand that rotations, reflections and translations preserve length and angle so that a transformed shape under any of these transformations remains congruent to the original shape |
| 5.2J | understand that enlargements are specified by a centre and a scale factor |
| 5.2K | understand that enlargements preserve angles and not lengths |
| 5.2L | enlarge a shape given the scale factor |
| 5.2M | identify and give complete descriptions of transformations |

POSSIBLE SUCCESS CRITERIA

Understand that translations are specified by a distance and direction (using a vector).

Describe and transform a given shape by a reflection or a rotation or a translation.

Find the scale factor of an enlargement.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

COMMON MISCONCEPTIONS

The directions on a column vector often get mixed up.

Correct language must be used: students often use 'turn' rather than 'rotate'.

NOTES

Emphasise the need to describe the transformations fully, and if asked to describe a 'single' transformation they should not include two types.

It is essential to check the increments on the coordinate grid when translating shapes.

Students may need reminding about how to find the equations of straight lines, including those parallel to the axes.

When reflecting shapes, the students must include mirror lines on or through original shapes.

EXAMPLE QUESTIONS FROM SAMs: 1F q5d; 2F q21

25. Pythagoras' Theorem and Trigonometry**Teaching time**

11-13 hours

OBJECTIVES

| | |
|-------------|--|
| 4.8A | know, understand and use Pythagoras' Theorem in two dimensions |
| 4.8B | know, understand and use sine, cosine and tangent of acute angles to determine lengths and angles of a right-angled triangle |
| 4.8C | apply trigonometrical methods to solve problems in two dimensions |

POSSIBLE SUCCESS CRITERIA

Does 2, 3, 6 give a right angled triangle?

Justify when to use Pythagoras' Theorem and when to use trigonometry.

Find a given side or angle using trigonometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' Theorem or a combination of Pythagoras' Theorem and the trigonometric ratios.

In addition to abstract problems, students should be encouraged to apply Pythagoras' Theorem and/or the trigonometric ratios to real-life scenarios that require them to evaluate whether their answer fulfils certain criteria, e.g. the angle of elevation of 6.5 m ladder cannot exceed 65° . What is the greatest height it can reach?

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.

Students forget to square root their final answer or round their answer prematurely.

NOTES

Students may need reminding about surds.

Drawing the squares on the 3 sides will help to illustrate the theorem.

Include examples with triangles drawn in all four quadrants.

Scale drawings are not acceptable.

Calculators need to be in degree mode.

Use a suitable mnemonic to remember SOHCAHTOA.

Use Pythagoras' Theorem and trigonometry together.

EXAMPLE QUESTIONS FROM SAMs: 1F q22

26. Similarity and congruence in 2D**Teaching time**

4-6 hours

OBJECTIVES

| | |
|--------------|---|
| 4.2F | understand congruence as meaning the same shape and size |
| 4.2G | understand that two or more polygons with the same shape and size are said to be congruent to each other |
| 4.11A | understand and use the geometrical properties that similar figures have corresponding lengths in the same ratio but corresponding angles remain unchanged |

POSSIBLE SUCCESS CRITERIA

Understand similarity as one shape being an enlargement of the other.

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not equal in size.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using scale diagrams, including bearings and maps, provides a rich source of real-life examples and links to other areas of mathematics.

COMMON MISCONCEPTIONS

Students may incorrectly believe that all polygons are regular or that all triangles have a rotational symmetry of order 3.

Often students think that when a shape is enlarged the angles also get bigger.

NOTES

Use simple scale factors that are easily calculated mentally to introduce similar shapes.

Reinforce the fact that the sizes of angles are maintained when a shape is enlarged.

EXAMPLE QUESTIONS FROM SAMs: 2F q6

27. Constructions and bearings**Teaching time**

3-5 hours

OBJECTIVES

| | |
|-------------|--|
| 4.5B | construct triangles and other two-dimensional shapes using a combination of a ruler, a protractor and compasses |
| 4.5D | use straight edge and compasses to: (i) construct the perpendicular bisector of a line segment (ii) construct the bisector of an angle |

POSSIBLE SUCCESS CRITERIA

Construct a given triangle.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Link problems with other areas of mathematics, such as the trigonometric ratios and Pythagoras' Theorem.

COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

NOTES

Drawings should be done in pencil.
Construction arcs should be left in.

EXAMPLE QUESTIONS FROM SAMs: 1F q20; 2F q6

28. Graphical representation of data**Teaching time**

6-8 hours

OBJECTIVES

| | |
|-------------|--|
| 6.1A | use different methods of presenting data |
| 6.1B | use appropriate methods of tabulation to enable the construction of statistical diagrams |
| 6.1C | interpret statistical diagrams |

POSSIBLE SUCCESS CRITERIA

Construct a frequency table.

Interpret and draw a pictogram.

Interpret and draw a bar chart.

From a simple pie chart identify the frequency represented by $\frac{1}{4}$ and $\frac{1}{2}$ sections.

Find the angle for one item.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to decide what the scales on any axis should be to be able to present information.

From inspection of a pie chart, students should be able to identify the fraction of the total represented and know when that total can be calculated and compared with another pie chart

COMMON MISCONCEPTIONS

Students struggle to make the link between what the data in a frequency table represents, so for example may state the 'frequency' rather than the interval when asked for the modal group.

In a pie chart, same size sectors for different sized data sets represent the same number rather than the same proportion.

NOTES

Ensure that you include a variety of scales, including decimal numbers of millions and thousands, time scales in hours, minutes, seconds.

Relate $\frac{1}{4}$, $\frac{1}{2}$, etc to percentages.

Practise dividing by 20, 30, 40, 60, etc.

Compare pie charts to identify similarities and differences.

Angles when drawing pie charts should be accurate to 2° .

EXAMPLE QUESTIONS FROM SAMs: 1F q3; 2F q11

29. Statistical measures**Teaching time**

6-8 hours

OBJECTIVES

| | |
|-------------|--|
| 6.2A | understand the concept of average |
| 6.2B | calculate the mean, median, mode and range for a discrete data set |
| 6.2C | calculate an estimate for the mean for grouped data |
| 6.2D | identify the modal class for grouped data |

POSSIBLE SUCCESS CRITERIA

State the median, mode, mean and range from a small data set.

Estimate the mean from a table of grouped and from a table of ungrouped data.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide a correct solution as a counter-argument to statements involving the “averages”, e.g. Susan states that the median is 15, she is wrong. Explain why.

Given the mean, median and mode of five positive whole numbers, can you find the numbers?

COMMON MISCONCEPTIONS

Often the $\Sigma(m \times f)$ is divided by the number of classes rather than Σf when estimating the mean.

NOTES

Encourage students to cross out the midpoints of each group once they have used these numbers to in $m \times f$. This helps students to avoid summing m instead of f .

Remind students how to find the midpoint of two numbers.

EXAMPLE QUESTIONS FROM SAMs: 1F q18; 2F q22

30. Probability**Teaching time**

8-10 hours

OBJECTIVES

| | |
|-------------|---|
| 6.3A | understand the language of probability |
| 6.3B | understand and use the probability scale |
| 6.3C | understand and use estimates or measures of probability from theoretical models |
| 6.3D | find probabilities from a Venn diagram |
| 6.3E | understand the concepts of a sample space and an event, and how the probability of an event happening can be determined from the sample space |
| 6.3F | list all the outcomes for single events and for two successive events in a systematic way |
| 6.3G | estimate probabilities from previously collected data |
| 6.3H | calculate the probability of the complement of an event happening |
| 6.3I | use the addition rule of probability for mutually exclusive events |
| 6.3J | understand and use the term 'expected frequency' |

POSSIBLE SUCCESS CRITERIA

Mark events on a probability scale and use the language of probability.

If the probability of outcomes are x , $2x$, $4x$, $3x$ calculate x .

Calculate the probability of an event from a frequency table.

Decide if a coin, spinner or game is fair.

Understand the use of the 0–1 scale to measure probability.

Know and apply the fact that the sum of probabilities for all outcomes is 1.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Lotteries provides a real life link to probability. Work out the probabilities of winning on different lotteries.

Students should be given the opportunity to justify the probability of events happening or not happening.

COMMON MISCONCEPTIONS

Not using fractions or decimals or percentages when giving an answer as a probability.

NOTES

Use this as an opportunity for practical work.

EXAMPLE QUESTIONS FROM SAMs: 1F q13, q18c; 2F q2, q4

International GCSE Mathematics
(Specification A)
(4MA1)
Higher Tier

Scheme of Work

| Unit number | | Title | Estimated teaching hours |
|--------------------------|----|--|--------------------------|
| Number | 1 | Decimals | 4 |
| | 2 | Special numbers, powers and roots | 6 |
| | 3 | Fractions | 4 |
| | 4 | Percentages | 5 |
| | 5 | Ratio and proportion | 3 |
| | 6 | Indices and standard form | 4 |
| | 7 | Degree of accuracy | 4 |
| | 8 | Set language, notation and Venn diagrams | 6 |
| Algebra | 9 | Algebraic manipulation | 8 |
| | 10 | Expressions, formulae and rearranging formulae | 6 |
| | 11 | Linear equations and inequalities | 4 |
| | 12 | Sequences | 4 |
| | 13 | Real life graphs | 2 |
| | 14 | Linear graphs | 7 |
| | 15 | Quadratic equations and graphs | 8 |
| | 16 | Harder graphs and transformation of graphs | 7 |
| | 17 | Simultaneous equations | 5 |
| | 18 | Function notation | 7 |
| | 19 | Calculus | 8 |
| Space, shape and measure | 20 | Compound measures | 5 |
| | 21 | Geometry of shapes | 6 |
| | 22 | Constructions and bearings | 4 |
| | 23 | Perimeter, area and volume | 8 |
| | 24 | Pythagoras' theorem and Trigonometry | 8 |
| | 25 | Transformations | 5 |
| | 26 | Circle properties | 6 |
| | 27 | Advanced trigonometry | 8 |
| | 28 | Similar shapes | 7 |
| | 29 | Vectors | 6 |
| Handling Data | 30 | Graphical representation of data | 5 |
| | 31 | Statistical measures | 4 |
| | 32 | Probability | 6 |
| | | Total | 180 |

OBJECTIVES / SPECIFICATION REFERENCES

| Unit and title | | Specification Reference | | Est teaching hours |
|----------------|----------------------------|-------------------------|---|--------------------|
| | | Fnd | Higher | |
| 1 | Decimals | 1.3A | convert recurring decimals into fractions | 4 |
| | | 1.8B | round to a given number of significant figures or decimal places | |
| | | 1.8D | use estimation to evaluate approximations to numerical calculations | |
| | | 1.11A | use a scientific electronic calculator to determine numerical results | |
| 2 | Special numbers and powers | 1.4D | express integers as product of powers of prime factors | 6 |
| | | 1.4E | find highest common factors (HCF) and lowest common multiples (LCM) | |
| | | 1.4A | understand the meaning of surds | |
| | | 1.4B | manipulate surds, including rationalising a denominator | |
| | | 1.4C | use index laws to simplify and evaluate numerical expressions involving integer, fractional and negative powers | |
| 3 | Fractions | 1.2D | order fractions and calculate a given fraction of a given quantity | 4 |
| | | 1.2E | express a given number as a fraction of another number | |
| | | 1.2G | convert a fraction to a decimal or percentage | |
| | | 1.2F | use common denominators to add and subtract fractions and mixed numbers | |
| | | 1.2H | understand and use fractions as multiplicative inverses | |
| | | 1.2I | multiply and divide fractions and mixed numbers | |
| 4 | Percentages | 1.6B | express a given number as a percentage of another number; | 5 |
| | | 1.6C | express a percentage as a fraction and as a decimal | |
| | | 1.6D | understand the multiplicative nature of percentages as operators | |
| | | 1.6E | solve simple percentage problems, including percentage increase and decrease | |
| | | 1.6F | use reverse percentages | |
| | | 1.6G | use compound interest and depreciation | |
| | | 1.6A | use repeated percentage change | |
| | | 1.6B | solve compound interest problems | |

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| 5 | Ratio and proportion | 1.7A | use ratio notation, including reduction to its simplest form and its various links to fraction notation | 3 |
| | | 1.7B | divide a quantity in a given ratio or ratios | |
| | | 1.7C | use the process of proportionality to evaluate unknown quantities | |
| | | 1.7D | calculate an unknown quantity from quantities that vary in direct proportion | |
| | | 1.7E | solve word problems about ratio and proportion | |
| | | 1.10A | use and apply number in everyday personal, domestic or community life | |
| | | 1.10B | carry out calculations using standard units of mass, length, area, volume and capacity | |
| | | 1.10C | understand and carry out calculations using time, and carry out calculations using money, including converting between currencies | |
| 6 | Indices and standard form | 1.4C | use index notation and index laws for multiplication and division of positive and negative integer powers including zero | 4 |
| | | 1.9A | calculate with and interpret numbers in the form $a \times 10^n$ where n is an integer and $1 \leq a < 10$ | |
| | | 1.9A | solve problems involving standard form | |
| 7 | Degree of accuracy | 1.8C | identify upper and lower bounds where values are given to a degree of accuracy | 4 |
| | | 1.8A | solve problems using upper and lower bounds where values are given to a degree of accuracy | |
| 8 | Set language, notation and Venn diagrams | 1.5A | understand the definition of a set | 6 |
| | | 1.5B | use the set notation \cup , \cap and \in and \notin | |
| | | 1.5C | understand the concept of the universal set and the empty set and the symbols for these sets | |
| | | 1.5D | understand and use the complement of a set | |
| | | 1.5E | use Venn diagrams to represent sets | |
| | | 6.3D | find probabilities from a Venn diagram | |
| | | 1.5A | understand sets defined in algebraic terms, and understand and use subsets | |
| | | 1.5B | use Venn diagrams to represent sets and the number of elements in sets | |
| | | 1.5C | use the notation $n(A)$ for the number of elements in the set A | |
| | | 1.5D | use sets in practical situations | |

Algebra : Units 9 – 19

OBJECTIVES / SPECIFICATION REFERENCES

| Unit and title | | Specification Reference | | Est teaching hours |
|----------------|--|-------------------------|--|--------------------|
| | | Fnd | Higher | |
| 9 | Algebraic manipulation | 2.1A | use index notation involving fractional, negative and zero powers | 8 |
| | | 2.1D | use index laws in simple cases | |
| | | 2.2B | collect like terms | |
| | | 2.2C | multiply a single term over a bracket | |
| | | 2.2D | take out common factors | |
| | | 2.2A | expand the product of two or more linear expressions | |
| | | 2.2B | understand the concept of a quadratic expression and be able to factorise such expressions | |
| | | 2.2C | manipulate algebraic fractions where the numerator and/or the denominator can be numeric, linear or quadratic | |
| | | 2.2D | complete the square for a given quadratic expression | |
| | | 2.2E | use algebra to support and construct proofs | |
| 10 | Expressions, formulae and rearranging formulae | 2.3C | substitute positive and negative integers, decimals and fractions for words and letters in expressions and formulae | 6 |
| | | 2.3D | use formulae from mathematics and other real-life contexts expressed initially in words or diagrammatic form and convert to letters and symbols | |
| | | 2.3E | derive a formula or expression | |
| | | 2.3A | understand the process of manipulating formulae or equations to change the subject, to include cases where the subject may appear twice or a power of the subject occurs | |
| | | 2.5A | set up problems involving direct or inverse proportion and relate algebraic solutions to graphical representation of the equations | |
| 11 | Linear equations and inequalities | 2.4A | solve linear equations, with integer or fractional coefficients, in one unknown in which the unknown appears on either side or both sides of the equation | 4 |
| | | 2.4B | set up simple linear equations from given data | |
| | | 2.8C | solve simple linear inequalities in one variable and represent the solution set on a number line | |

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|----|--|-------------|---|---|
| 12 | Sequences | 3.1A | understand and use common difference (d) and first term (a) in an arithmetic sequence | 4 |
| | | 3.1B | know and use n th term $= a + (n - 1)d$ | |
| | | 3.1C | find the sum of the first n terms of an arithmetic series (S_n) | |
| 13 | Real life graphs | 3.3A | interpret information presented in a range of linear and non-linear graphs | 2 |
| 14 | Linear graphs | 3.3E | determine the coordinates of the midpoint of a line segment, given the coordinates of the two end points | 7 |
| | | 3.3G | find the gradient of a straight line | |
| | | 3.3H | recognize that equations of the form $y = mx + c$ are straight line graphs with gradient m and intercept on the y -axis at the point $(0, c)$ | |
| | | 3.3I | recognize, generate points and plot graphs of linear functions | |
| | | 3.3F | calculate the gradient of a straight line given the coordinates of two points | |
| | | 3.3G | find the equation of a straight line parallel to a given line; find the equation of a straight line perpendicular to a given line | |
| | | 2.8D | represent simple linear inequalities on rectangular Cartesian graphs | |
| | | 2.8E | identify regions on rectangular Cartesian graphs defined by simple linear inequalities | |
| 15 | Quadratic equations, inequalities and graphs | 2.8B | identify harder examples of regions defined by linear inequalities | 8 |
| | | 2.7A | solve quadratic equations by factorization | |
| | | 2.7B | solve quadratic equations by using the quadratic formula or completing the square | |
| | | 2.7C | form and solve quadratic equations from data given in a context | |
| | | 2.8A | solve quadratic inequalities in one unknown and represent the solution set on a number line | |
| | | 3.3I | recognize, generate points and plot graphs of quadratic functions | |

| | | | |
|----|--|--|---|
| 16 | Harder graphs and transformation of graphs | <p>3.3A recognise, plot and draw graphs with equation:</p> $y = Ax^3 + Bx^2 + Cx + D$ <p>in which:</p> <p>(i) the constants are integers and some could be zero</p> <p>(ii) the letters x and y can be replaced with any other two letters or:</p> $y = Ax^3 + Bx^2 + Cx + D + \frac{E}{x} + \frac{F}{x^2}$ <p>in which:</p> <p>(i) the constants are numerical and at least three of them are zero</p> <p>(ii) the letters x and y can be replaced with any other two letters or:</p> <p>$y = \sin x, y = \cos x, y = \tan x$ for angles of any size (in degrees)</p> | 7 |
| | | <p>3.3B apply to the graph of $y = f(x)$ the transformations $y = f(x) + a, y = f(ax), y = f(x + a), y = af(x)$ for linear, quadratic, sine and cosine functions</p> | |
| | | <p>3.3C interpret and analyse transformations of functions and write the functions algebraically</p> | |
| | | <p>3.3D find the gradients of non-linear graphs</p> | |
| | | <p>3.3E find the intersection points of two graphs, one linear (y_1) and one non-linear (y_2), and recognise that the solutions correspond to the solutions of $y_2 - y_1 = 0$</p> | |
| 17 | Simultaneous equations | <p>2.6A calculate the exact solution of two simultaneous equations in two unknowns</p> | 5 |
| | | <p>2.6B interpret the equations as lines and the common solution as the point of intersection</p> | |
| | | <p>2.7D solve simultaneous equations in two unknowns, one equation being linear and the other being quadratic</p> | |
| 18 | Function notation | <p>3.2A understand the concept that a function is a mapping between elements of two sets</p> | 7 |
| | | <p>3.2B use function notations of the form $f(x) = \dots$ and $f : x \rightarrow \dots$</p> | |
| | | <p>3.2C understand the terms 'domain' and 'range' and which values may need to be excluded from a domain</p> | |
| | | <p>3.2D understand and find the composite function fg and the inverse function f^{-1}</p> | |
| 19 | Calculus | <p>3.4A understand the concept of a variable rate of change</p> | 8 |
| | | <p>3.4B differentiate integer powers of x</p> | |
| | | <p>3.4C determine gradients, rates of change, stationary points, turning points (maxima and minima) by differentiation and relate these to graphs</p> | |
| | | <p>3.4D distinguish between maxima and minima by considering the general shape of the graph only</p> | |

Higher tier

| | | | |
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| | | 3.4E apply calculus to linear kinematics and to other simple practical problems | |
|--|--|--|--|

Shape, space and measure : Units 20 – 29

OBJECTIVES / SPECIFICATION REFERENCES

| Unit and title | | Specification Reference | | Est teaching hours |
|----------------|----------------------------|-------------------------|--|--------------------|
| | | Fnd | Higher | |
| 20 | Compound measures | 4.4G | use compound measure such as speed, density and pressure | 5 |
| | | 4.9A | convert measurements within the metric system to include linear and area units | |
| | | 4.10A | convert between units of volume within the metric system | |
| 21 | Geometry of shapes | 4.1B | use angle properties of intersecting lines, parallel lines and angles on a straight line | 6 |
| | | 4.1D | understand the terms 'isosceles', 'equilateral' and 'right-angled triangles' and the angle properties of these triangles | |
| | | 4.2B | understand and use the term 'quadrilateral' and the angle sum property of quadrilaterals | |
| | | 4.2C | understand and use the properties of the parallelogram, rectangle, square, rhombus, trapezium and kite | |
| | | 4.2D | understand the term 'regular polygon' and calculate interior and exterior angles of regular polygons | |
| | | 4.2E | understand and use the angle sum of polygons | |
| | | 4.7A | provide reasons, using standard geometrical statements, to support numerical values for angles obtained in any geometrical context involving lines, polygons and circles | |
| 22 | Constructions and bearings | 4.5B | construct triangles and other two-dimensional shapes using a combination of a ruler, a protractor and compasses | 4 |
| | | 4.5D | use straight edge and compasses to: (i) construct the perpendicular bisector of a line segment (ii) construct the bisector of an angle | |
| | | 4.4D | understand angle measure including three-figure bearings | |
| | | 4.5C | solve problems using scale drawings | |
| | | 4.11B | use and interpret maps and scale drawings | |
| 23 | Perimeter, area and volume | 4.9B | find the perimeter of shapes made from triangles and rectangles | 8 |
| | | 4.9C | find the area of simple shapes using the formulae for the areas of triangles and rectangles | |
| | | 4.9D | find the area of parallelograms and trapezia | |
| | | 4.9A | find perimeters and areas of sectors of circles | |
| | | 4.10C | find the surface area of simple shapes using the area formulae for triangles and rectangles | |
| | | 4.10D | find the surface area of a cylinder | |

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|----|--------------------------------------|--------------|--|---|
| | | 4.10E | find the volume of prisms, including cuboids and cylinders, using an appropriate formula | |
| | | 4.10A | find the surface area and volume of a sphere and a right circular cone using relevant formulae | |
| 24 | Pythagoras' theorem and Trigonometry | 4.8A | know, understand and use Pythagoras' Theorem in two dimensions | 8 |
| | | 4.8B | know, understand and use sine, cosine and tangent of acute angles to determine lengths and angles of a right-angled triangle | |
| | | 4.8C | apply trigonometrical methods to solve problems in two dimensions | |
| | | 4.8A | understand and use sine, cosine and tangent of obtuse angles | |
| | | 4.8B | understand and use angles of elevation and depression | |
| 25 | Transformations | 5.2A | understand that rotations are specified by a centre and an angle | 5 |
| | | 5.2B | rotate a shape about a point through a given angle | |
| | | 5.2C | recognize that an anti-clockwise rotation is a <i>positive</i> angle of rotation and a clockwise rotation is a <i>negative</i> angle of rotation | |
| | | 5.2D | understand that reflections are specified by a mirror line | |
| | | 5.2E | construct a mirror line given an object and reflect a shape given a mirror line | |
| | | 5.2F | understand that translations are specified by a distance and direction | |
| | | 5.2G | translate a shape | |
| | | 5.2H | understand and use column vectors in translations | |
| | | 5.2I | understand that rotations, reflections and translations preserve length and angle so that a transformed shape under any of these transformations remains congruent to the original shape | |
| | | 5.2J | understand that enlargements are specified by a centre and a scale factor | |
| | | 5.2K | understand that enlargements preserve angles and not lengths | |
| | | 5.2L | enlarge a shape given the scale factor | |
| | | 5.2M | identify and give complete descriptions of transformations | |
| 26 | Circle properties | 4.6A | understand and use the internal and external intersecting chord properties | 6 |
| | | 4.6B | recognise the term 'cyclic quadrilateral' | |

| | | | |
|----|-----------------------|--|---|
| | | <p>4.6C understand and use angle properties of the circle including:</p> <ul style="list-style-type: none"> (i) angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the remaining part of the circumference (ii) angle subtended at the circumference by a diameter is a right angle (iii) angles in the same segment are equal (iv) the sum of the opposite angles of a cyclic quadrilateral is 180° (v) the alternate segment theorem | |
| 27 | Advanced trigonometry | 4.8C understand and use the sine and cosine rules for any triangle | 8 |
| | | 4.8D use Pythagoras' theorem in three dimensions | |
| | | 4.8E understand and use the formula $\frac{1}{2}ab \sin C$ for the area of a triangle | |
| | | 4.8F apply trigonometrical methods to solve problems in three dimensions, including finding the angle between a line and a plane | |
| 28 | Similar shapes | 4.2F understand congruence as meaning the same shape and size | 7 |
| | | 4.2G understand that two or more polygons with the same shape and size are said to be congruent to each other | |
| | | 4.11A understand and use the geometrical properties that similar figures have corresponding lengths in the same ratio but corresponding angles remain unchanged | |
| | | 4.11A understand that areas of similar figures are in the ratio of the square of corresponding sides | |
| | | 4.11B understand that volumes of similar figures are in the ratio of the cube of corresponding sides | |
| | | 4.11C use areas and volumes of similar figures in solving problems | |
| 29 | Vectors | 5.1A understand that a vector has both magnitude and direction | 6 |
| | | 5.1B understand and use vector notation including column vectors | |
| | | 5.1C multiply vectors by scalar quantities | |
| | | 5.1D add and subtract vectors | |
| | | 5.1E calculate the modulus (magnitude) of a vector | |
| | | 5.1F find the resultant of two or more vectors | |
| | | 5.1G apply vector methods for simple geometrical proofs | |

Handling Data : Units 30 – 32

OBJECTIVES / SPECIFICATION REFERENCES

| Unit and title | | Specification Reference | | Est teaching hours |
|----------------|----------------------------------|-------------------------|---|--------------------|
| | | Fnd | Higher | |
| 30 | Graphical representation of data | 6.1A | construct and interpret histograms | 5 |
| | | 6.1B | construct cumulative frequency diagrams from tabulated data | |
| | | 6.1C | use cumulative frequency diagrams | |
| 31 | Statistical measures | 6.2A | understand the concept of average | 4 |
| | | 6.2B | calculate the mean, median, mode and range for a discrete data set | |
| | | 6.2C | calculate an estimate for the mean for grouped data | |
| | | 6.2D | identify the modal class for grouped data | |
| | | 6.2A | estimate the median from a cumulative frequency diagram | |
| | | 6.2B | understand the concept of a measure of spread | |
| | | 6.2C | find the interquartile range from a discrete data set | |
| | | 6.2D | estimate the interquartile range from a cumulative frequency diagram | |
| 32 | Probability | 6.3C | understand and use estimates or measures of probability from theoretical models | 6 |
| | | 6.3D | find probabilities from a Venn diagram | |
| | | 6.3E | understand the concepts of a sample space and an event, and how the probability of an event happening can be determined from the sample space | |
| | | 6.3G | estimate probabilities from previously collected data | |
| | | 6.3H | calculate the probability of the complement of an event happening | |
| | | 6.3I | use the addition rule of probability for mutually exclusive events | |
| | | 6.3J | understand and use the term 'expected frequency' | |
| | | 6.3A | draw and use tree diagrams | |
| | | 6.3B | determine the probability that two or more independent events will occur | |
| | | 6.3C | use simple conditional probability when combining events | |
| | | 6.3D | apply probability to simple problems | |

It is assumed that students being prepared for the Higher tier will have knowledge of the Foundation tier content.

1. Decimals

Teaching time
3-5 hours

OBJECTIVES

| | |
|---------------|---|
| H1.3A | convert recurring decimals into fractions |
| F1.8B | round to a given number of significant figures or decimal places |
| F1.8D | use estimation to evaluate approximations to numerical calculations |
| F1.11A | use a scientific electronic calculator to determine numerical results |

POSSIBLE SUCCESS CRITERIA

Estimate the value of $\frac{34.5 \times 7.34}{0.154}$

Change $0.4\dot{5}$ into a fraction in its simplest form.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Use of decimals within a problem.

Show algebraically that $3.0\dot{1}$ can be written as $3\frac{1}{90}$

Links with other areas of Mathematics can be made by using surds in Pythagoras and when using trigonometric ratios.

COMMON MISCONCEPTIONS

Significant figure and decimal place rounding are often confused.
Some pupils may think $35\ 934 = 36$ to two significant figures.

NOTES

The expectation for Higher tier is that much of this work will be reinforced throughout the course.

Make sure students are absolutely clear about the difference between significant figures and decimal places.

EXAMPLE QUESTIONS FROM SAMs: -

2. Special numbers and powers**Teaching time**

5-7 hours

OBJECTIVES

| | |
|--------------|---|
| F1.4D | express integers as product of powers of prime factors |
| F1.4E | find highest common factors (HCF) and lowest common multiples (LCM) |
| H1.4A | understand the meaning of surds |
| H1.4B | manipulate surds, including rationalising a denominator |
| H1.4C | use index laws to simplify and evaluate numerical expressions involving integer, fractional and negative powers |

POSSIBLE SUCCESS CRITERIA

What is the value of 2^5 ?

Find the HCF and LCM of 12 and 20

Write a number as a product of its prime factors.

Prove that the square root of 45 lies between 6 and 7.

Simplify $\sqrt{40}$

Rationalise the denominator of $\frac{5}{\sqrt{10}}$; $\frac{6}{1+\sqrt{2}}$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that use indices instead of integers will provide rich opportunities to apply the knowledge in this unit in other areas of Mathematics.

COMMON MISCONCEPTIONS

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.

NOTES

Students need to know how to enter negative numbers into their calculator.

Use negative number and not minus number to avoid confusion with calculations.

Students need to be encouraged to learn squares from 2×2 to 15×15 and cubes of 2, 3, 4, 5 and 10, and corresponding square and cube roots.

EXAMPLE QUESTIONS FROM SAMs: 4H q1, q24

3. Fractions**Teaching time**

3-5 hours

OBJECTIVES

| | |
|--------------|---|
| F1.2D | order fractions and calculate a given fraction of a given quantity |
| F1.2E | express a given number as a fraction of another number |
| F1.2G | convert a fraction to a decimal or percentage |
| F1.2F | use common denominators to add and subtract fractions and mixed numbers |
| F1.2H | understand and use fractions as multiplicative inverses |
| F1.2I | multiply and divide fractions and mixed numbers |

POSSIBLE SUCCESS CRITERIA

Express a given number as a fraction of another, including where the fraction is, for example, greater than 1, e.g. $\frac{120}{100} = 1\frac{2}{10} = 1\frac{1}{5}$.

Answer the following: James delivers 56 newspapers. $\frac{3}{8}$ of the newspapers have a magazine.

How many of the newspapers have a magazine?

Prove whether a fraction is terminating or recurring.

Convert a fraction to a decimal including where the fraction is greater than 1.

Convince me that 0.125 is $\frac{1}{8}$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many of these topics provide opportunities for reasoning in real-life contexts, particularly percentages:

Calculate original values and evaluate statements in relation to this value justifying which statement is correct.

COMMON MISCONCEPTIONS

The larger the denominator, the larger the fraction.

Incorrect links between fractions and decimals, such as thinking that $\frac{1}{5} = 0.15$, $5\% = 0.5$, $4\% = 0.4$, etc.

NOTES

Ensure that you include fractions where only one of the denominators needs to be changed, in addition to where both need to be changed for addition and subtraction.

Include multiplying and dividing integers by fractions.

Encourage use of the fraction button.

EXAMPLE QUESTIONS FROM SAMs: 3H q4; 4H q10

4. Percentages**Teaching time**

4-6 hours

OBJECTIVES

| | |
|--------------|--|
| F1.6B | express a given number as a percentage of another number; |
| F1.6C | express a percentage as a fraction and as a decimal |
| F1.6D | understand the multiplicative nature of percentages as operators |
| F1.6E | solve simple percentage problems, including percentage increase and decrease |
| F1.6F | use reverse percentages |
| F1.6G | use compound interest and depreciation |
| H1.6A | use repeated percentage change |
| H1.6B | solve compound interest problems |

POSSIBLE SUCCESS CRITERIA

Be able to work out the price of a deposit, given the price of a sofa is £480 and the deposit is 15% of the price, without a calculator.

Find fractional percentages of amounts, with and without using a calculator.

Work out 56 cm as a percentage of 2.5 m.

Work out the interest earned when £5600 is invested for 3 years at 2.5% compound interest.

Find the original price when the sale price of an item is £68 following a reduction of 15%

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many of these topics provide opportunities for reasoning in real-life contexts, particularly percentages:

Calculate original values and evaluate statements in relation to this value justifying which statement is correct.

COMMON MISCONCEPTIONS

Incorrect links between fractions and decimals, such as thinking that $\frac{1}{5} = 0.15$, $5\% = 0.5$,

$4\% = 0.4$, etc.

It is not possible to have a percentage greater than 100%.

NOTES

Students should be reminded of basic percentages.

Amounts of money should always be rounded to the nearest penny, except where successive calculations are done (i.e. compound interest, which is covered in a later unit).

Emphasise the use of percentages in real-life situations.

EXAMPLE QUESTIONS FROM SAMs: 3H q4, q8; 4H q5, q8

5. Ratio and proportion**Teaching time**

2-4 hours

OBJECTIVES

| | |
|---------------|---|
| F1.7A | use ratio notation, including reduction to its simplest form and its various links to fraction notation |
| F1.7B | divide a quantity in a given ratio or ratios |
| F1.7C | use the process of proportionality to evaluate unknown quantities |
| F1.7D | calculate an unknown quantity from quantities that vary in direct proportion |
| F1.7E | solve word problems about ratio and proportion |
| F1.10A | use and apply number in everyday personal, domestic or community life |
| F1.10B | carry out calculations using standard units of mass, length, area, volume and capacity |
| F1.10C | understand and carry out calculations using time, and carry out calculations using money, including converting between currencies |

POSSIBLE SUCCESS CRITERIA

Write/interpret a ratio to describe a situation such as 1 blue for every 2 red ..., 3 adults for every 10 children ...

Recognise that two paints mixed red to yellow 5 : 4 and 20 : 16 are the same colour.

When a quantity is split in the ratio 3:5, what fraction does each person get?

Find amounts for three people when amount for one given.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving sharing in a ratio that include percentages rather than specific numbers such can provide links with other areas of Mathematics:

In a youth club the ratio of the number of boys to the number of girls is 3 : 2 . 30% of the boys are under the age of 14 and 60% of the girls are under the age of 14. What percentage of the youth club is under the age of 14?

COMMON MISCONCEPTIONS

Students often identify a ratio-style problem and then divide by the number given in the question, without fully understanding the question.

NOTES

Three-part ratios are usually difficult for students to understand.

Also include using decimals to find quantities.

Use a variety of measures in ratio and proportion problems.

EXAMPLE QUESTIONS FROM SAMs: 3H q2, q9d

6. Indices and standard form**Teaching time**

3-5 hours

OBJECTIVES

| | |
|--------------|--|
| F1.4C | use index notation and index laws for multiplication and division of positive and negative integer powers including zero |
| F1.9A | calculate with and interpret numbers in the form $a \times 10^n$ where n is an integer and $1 \leq a < 10$ |
| H1.9A | solve problems involving standard form |

POSSIBLE SUCCESS CRITERIA

Evaluate $(2^3 \times 2^5) \div 2^4$, 4^0 , $8^{\frac{2}{3}}$.

Work out the value of n in $40 = 5 \times 2^n$.

Write 51080 in standard form.

Write 3.74×10^{-6} as an ordinary number.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

COMMON MISCONCEPTIONS

Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form.

NOTES

Standard form is used in science and there are lots of cross-curricular opportunities. Students need to be provided with plenty of practice in using standard form with calculators.

EXAMPLE QUESTIONS FROM SAMs: 3H q9, 4H 4d

7. Degree of accuracy**Teaching time**

3-5 hours

OBJECTIVES

| | |
|--------------|--|
| F1.8C | identify upper and lower bounds where values are given to a degree of accuracy |
| H1.8A | solve problems using upper and lower bounds where values are given to a degree of accuracy |

POSSIBLE SUCCESS CRITERIA

Round 16,000 people to the nearest 1000.

Round 1100 g to 1 significant figure.

Work out the upper and lower bounds of a formula where all terms are given to 1 decimal place.

Be able to justify that measurements to the nearest whole unit may be inaccurate by up to one half in either direction.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This sub-unit provides many opportunities for students to evaluate their answers and provide counter-arguments in mathematical and real-life contexts, in addition to requiring them to understand the implications of rounding their answers.

COMMON MISCONCEPTIONS

Students readily accept the rounding for lower bounds, but take some convincing in relation to upper bounds.

NOTES

Students should use 'half a unit above' and 'half a unit below' to find upper and lower bounds. Encourage use a number line when introducing the concept.

EXAMPLE QUESTIONS FROM SAMs: 3H q17

8. Set language, notation and Venn diagrams**Teaching time**

5-7 hours

OBJECTIVES

| | |
|--------------|--|
| F1.5A | understand the definition of a set |
| F1.5B | use the set notation \cup , \cap and \in and \notin |
| F1.5C | understand the concept of the universal set and the empty set and the symbols for these sets |
| F1.5D | understand and use the complement of a set |
| F1.5E | use Venn diagrams to represent sets |
| F6.3D | find probabilities from a Venn diagram |
| H1.5A | understand sets defined in algebraic terms, and understand and use subsets |
| H1.5B | use Venn diagrams to represent sets and the number of elements in sets |
| H1.5C | use the notation $n(A)$ for the number of elements in the set A |
| H1.5D | use sets in practical situations |

POSSIBLE SUCCESS CRITERIA

Universal set is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 2, 3, 4, 5, 6\}$, $B = \{2, 4, 6, 8\}$; Write down $A \cap B$, $A \cup B$

$C = \{1, 3, 5\}$; write down C'

Is $4 \in C$, is $4 \in A$, is C a subset of A ?

Find $n(A)$.

Draw a Venn diagram to show the universal set, A , B and C ;

If a number is picked at random, find $P(A \cap B)$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given Universal set is $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{5, 7, 9\}$ and $B = \{1, 3, 5, 7\}$

Write down a possible set C so that $A \cap C = \{7\}$ and C has 4 members

COMMON MISCONCEPTIONS

$A = \{5, 7, 9\}$ and $B = \{1, 3, 5, 7\}$ then $A \cup B = \{1, 3, 5, 5, 7, 7, 9\}$

NOTES

When drawing a Venn diagram it is a good idea to put members in the intersection first.

EXAMPLE QUESTIONS FROM SAMs: 4H q15

9. Algebraic manipulation**Teaching time**

7-9 hours

OBJECTIVES

| | |
|--------------|---|
| H2.1A | use index notation involving fractional, negative and zero powers |
| F2.1D | use index laws in simple cases |
| F2.2B | collect like terms |
| F2.2C | multiply a single term over a bracket |
| F2.2D | take out common factors |
| H2.2A | expand the product of two or more linear expressions |
| H2.2B | understand the concept of a quadratic expression and be able to factorise such expressions |
| H2.2C | manipulate algebraic fractions where the numerator and/or the denominator can be numeric, linear or quadratic |
| H2.2D | complete the square for a given quadratic expression |
| H2.2E | use algebra to support and construct proofs |

POSSIBLE SUCCESS CRITERIA

Simplify $4p - 2q^2 + 1 - 3p + 5q^2$.

Simplify $z^4 \times z^3$, $y^3 \div y^2$, $(a^7)^2$, $(8x^6y^4)^{\frac{1}{3}}$.

Factorise $15x^2y - 35x^2y^2$; $6x^2 - 7x + 1$.

Expand and simplify $3(t - 1) + 57$; $(3x + 2)(4x - 1)$; $(x + 7)(x - 1)(2x + 1)$

Use fractions when working in algebraic situations.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

COMMON MISCONCEPTIONS

When expanding two linear expressions, poor number skills involving negatives and times tables will become evident.

NOTES

Some of this will be a reminder from Key Stage 3 and could be introduced through Students will be asked to show 'algebraic working' when solving equations. Solutions with no working will score no marks.

Students can leave their answer in fraction form where appropriate. Emphasise that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

EXAMPLE QUESTIONS FROM SAMs: 3H q6a, q11, q15a, q18; 4H q4abc, q22

10. Expressions, formulae and rearranging equations**Teaching time**

5-7 hours

OBJECTIVES

| | |
|--------------|--|
| F2.3C | substitute positive and negative integers, decimals and fractions for words and letters in expressions and formulae |
| F2.3D | use formulae from mathematics and other real-life contexts expressed initially in words or diagrammatic form and convert to letters and symbols |
| F2.3E | derive a formula or expression |
| H2.3A | understand the process of manipulating formulae or equations to change the subject, to include cases where the subject may appear twice or a power of the subject occurs |
| H2.5A | set up problems involving direct or inverse proportion and relate algebraic solutions to graphical representation of the equations |

POSSIBLE SUCCESS CRITERIA

Find the value of $3x^2 - 2x$ for different values of x .

Find the value a in $v^2 = u^2 + 2as$ given values of the other variables.

Make a the subject of $v^2 = u^2 + 2as$

Make y the subject of $t = \sqrt{\frac{2-3y}{4}}$

Make t the subject of $a = \frac{2t+b}{3-t}$

Given that y is inversely proportional to x^2 and that when $x = 2$, $y = 3$ find a formula for y in terms of x .

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Justify and infer relationships in real-life scenarios to direct and inverse proportion such as ice cream sales and sunshine.

COMMON MISCONCEPTIONS

Confusing direct and inverse proportion.

NOTES

Students should be reminded to show all stages in their working.

Consider using science contexts for problems involving inverse proportionality, e.g. volume of gas inversely proportional to the pressure or frequency is inversely proportional to wavelength.

EXAMPLE QUESTIONS FROM SAMs: 4H q13, q16

11. Linear equations and inequalities**Teaching time**

5-7 hours

OBJECTIVES

| | |
|--------------|---|
| F2.4A | solve linear equations, with integer or fractional coefficients, in one unknown in which the unknown appears on either side or both sides of the equation |
| F2.4B | set up simple linear equations from given data |
| F2.8C | solve simple linear inequalities in one variable and represent the solution set on a number line |

POSSIBLE SUCCESS CRITERIASolve $5(x + 3) = 2x - 7$

Use inequality symbols to compare numbers.

Given a list of numbers, represent them on a number line using the correct notation.

Solve equations involving inequalities.

Solve $4x + 5 > x + 1$ Solve $\frac{2x-1}{3} - \frac{x+1}{2} = 5$ **OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Problems that require student to justify why certain values in a solution can be ignored.

Set up and solve problems involving linear equations.

COMMON MISCONCEPTIONS

When solving inequalities students often state their final answer as a number quantity, and exclude the inequality or change it to =.

Some students believe that -6 is greater than -3 .When solving equations like $\frac{2x-1}{3} - \frac{x+1}{2} = 5$ the common error is to forget to use the negative sign when expanding brackets.**NOTES**

Emphasise the importance of leaving their answer as an inequality (and not changing it to =).

Students can leave their answers in fractional form where appropriate.

Ensure that correct language is used to avoid reinforcing misconceptions: for example, 0.15 should never be read as 'zero point fifteen', and $5 > 3$ should be read as 'five is greater than 3', not '5 is bigger than 3'.**EXAMPLE QUESTIONS FROM SAMs: 4H q4e**

12. Sequences**Teaching time**

3-5 hours

OBJECTIVES

| | |
|--------------|---|
| H3.1A | understand and use common difference (d) and first term (a) in an arithmetic sequence |
| H3.1B | know and use n th term = $a + (n - 1)d$ |
| H3.1C | find the sum of the first n terms of an arithmetic series (S_n) |

POSSIBLE SUCCESS CRITERIA

Given a sequence, 'which is the 1st term greater than 50?'

Given the sequence 12, 7, 2, -3... find an expression in terms of n for the n th term.

Be able to solve problems involving sequences from real-life situations, such as:

- What is the amount of money after x months saving the same amount, or the height of tree that grows 6 m per year;

Given the sequence 5, 8, 11, 14... find the 50th term, the sum of the first 50 terms

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements about whether or not specific numbers or patterns are in a sequence and justify the reasons.

COMMON MISCONCEPTIONS

Students struggle to relate the position of the term to " n ".

Writing $n + 3$ instead of $3n - 1$ for the n th term of 2, 5, 8, 11...

NOTES

Emphasise use of $3n$ meaning $3 \times n$.

Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the n th term.

EXAMPLE QUESTIONS FROM SAMs: 3H q23; 4H q2

13. Real-life graphs**Teaching time**

3-4 hours

OBJECTIVES

| | |
|--------------|--|
| F3.3A | interpret information presented in a range of linear and non-linear graphs |
|--------------|--|

POSSIBLE SUCCESS CRITERIA

Interpret a description of a journey into a distance–time or speed–time graph.
Calculate various measures given a graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance graphs can provide opportunities for interpreting non-mathematical problems as a sequence of mathematical processes, whilst also requiring students to justify their reasons why one vehicle is faster than another.

COMMON MISCONCEPTIONS

Reading scales incorrectly is a common cause of errors.

NOTES

Careful annotation should be encouraged: it is good practice to label the axes and check that students understand the scales.

Use various measures in the distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours, and include large numbers in standard form.

Ensure that you include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.

EXAMPLE QUESTIONS FROM SAMs: -

14. Linear graphs**Teaching time**

6-8 hours

OBJECTIVES

| | |
|--------------|---|
| F3.3E | determine the coordinates of the midpoint of a line segment, given the coordinates of the two end points |
| F3.3G | find the gradient of a straight line |
| F3.3H | recognize that equations of the form $y = mx + c$ are straight line graphs with gradient m and intercept on the y -axis at the point $(0, c)$ |
| F3.3I | recognize, generate points and plot graphs of linear functions |
| H3.3F | calculate the gradient of a straight line given the coordinates of two points |
| H3.3G | find the equation of a straight line parallel to a given line; find the equation of a straight line perpendicular to a given line |
| F2.8D | represent simple linear inequalities on rectangular Cartesian graphs |
| F2.8E | identify regions on rectangular Cartesian graphs defined by simple linear inequalities |
| H2.8B | identify harder examples of regions defined by linear inequalities |

POSSIBLE SUCCESS CRITERIA

Find the equation of the line passing through two coordinates by calculating the gradient first.

Understand that the form $y = mx + c$ or $ax + by = c$ represents a straight line.

Show the region defined by $x < 3$, $y > 1$, $y < 3x + 2$

Find an equation of the line that goes through $(1, 2)$ and is parallel to $3y + 2x = 5$

Find an equation of the line that goes through $(1, 2)$ and is perpendicular to $3y + 2x = 5$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given an equation of a line provide a counter argument as to whether or not another equation of a line is parallel or perpendicular to the first line.

Decide if lines are parallel or perpendicular without drawing them and provide reasons.

COMMON MISCONCEPTIONS

Students can find visualisation of a question difficult, especially when dealing with gradients resulting from negative coordinates.

NOTES

Encourage students to sketch what information they are given in a question – emphasise that it is a sketch.

Careful annotation should be encouraged – it is good practice to label the axes and check that students understand the scales.

EXAMPLE QUESTIONS FROM SAMs: 3H q13; 4H q14

15. Quadratic equations, inequalities and graphs**Teaching time**

7-9 hours

OBJECTIVES

| | |
|--------------|---|
| H2.7A | solve quadratic equations by factorization |
| H2.7B | solve quadratic equations by using the quadratic formula or completing the square |
| H2.7C | form and solve quadratic equations from data given in a context |
| H2.8A | solve quadratic inequalities in one unknown and represent the solution set on a number line |
| F3.3I | recognize, generate points and plot graphs of quadratic functions |

POSSIBLE SUCCESS CRITERIASolve $3x^2 + 4 = 100$.Solve $2x^2 + 3x + 1 = 0$ Draw the graph of $y = x^2 + 5x + 6$

Know that the quadratic formula can be used to solve all quadratic equations, and often provides a more efficient method than factorising or completing the square.

Have an understanding of solutions that can be written in surd form.

Solve $x^2 < 9$; $2x^2 + 3x + 1 < 0$ **OPPORTUNITIES FOR REASONING/PROBLEM SOLVING**

Problems that require students to set up and solve a quadratic equation or inequality.

COMMON MISCONCEPTIONS

Using the formula involving negatives can result in incorrect answers.

All working must be shown when solving quadratic equations, including substitution into the quadratic formula.

NOTES

Remind students to use brackets for negative numbers when using a calculator, and remind them of the importance of knowing when to leave answers in surd form.

Reinforce the fact that some problems may produce one inappropriate solution which can be ignored.

Clear presentation of working out is essential.

Link with graphical representations.

EXAMPLE QUESTIONS FROM SAMs: 3H q6b, q15b

16. Harder graphs and transformation of graphs**Teaching time**

6-8 hours

OBJECTIVES

| | |
|--------------|---|
| H3.3A | <p>recognise, plot and draw graphs with equation:</p> $y = Ax^3 + Bx^2 + Cx + D$ <p>in which:</p> <p>(i) the constants are integers and some could be zero</p> <p>(ii) the letters x and y can be replaced with any other two letters or:</p> $y = Ax^3 + Bx^2 + Cx + D + \frac{E}{x} + \frac{F}{x^2}$ <p>in which:</p> <p>(i) the constants are numerical and at least three of them are zero</p> <p>(ii) the letters x and y can be replaced with any other two letters or:</p> $y = \sin x, y = \cos x, y = \tan x$ <p>for angles of any size (in degrees)</p> |
| H3.3B | <p>apply to the graph of $y = f(x)$ the transformations $y = f(x) + a$, $y = f(ax)$, $y = f(x + a)$, $y = af(x)$ for linear, quadratic, sine and cosine functions</p> |
| H3.3C | <p>interpret and analyse transformations of functions and write the functions algebraically</p> |
| H3.3D | <p>find the gradients of non-linear graphs</p> |
| H3.3E | <p>find the intersection points of two graphs, one linear (y_1) and one non-linear (y_2), and recognise that the solutions correspond to the solutions of $y_2 - y_1 = 0$</p> |

POSSIBLE SUCCESS CRITERIA

Select and use the correct mathematical techniques to draw graphs.

Identify a variety of functions by the shape of the graph.

Find the gradient, at a point, of a non-linear graph

Given the graph of $y = f(x)$, sketch the graph of $y = 2f(x)$; $y = f(x + 2)$; $y = -f(x)$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match equations of quadratics, cubics, reciprocal, trig functions with their graphs by recognising the shape or by sketching.

COMMON MISCONCEPTIONS

Students struggle with the concept of solutions and what they represent in concrete terms.

NOTES

Use lots of practical examples to help model the quadratic function, e.g. draw a graph to model the trajectory of a projectile and predict when/where it will land.

Ensure axes are labelled and pencils used for drawing.

Graphical calculations or appropriate ICT will allow students to see the impact of changing variables within a function.

EXAMPLE QUESTIONS FROM SAMs: 3H q20; 4H q19

17. Simultaneous equations**Teaching time**

4-6 hours

OBJECTIVES

| | |
|--------------|---|
| H2.6A | calculate the exact solution of two simultaneous equations in two unknowns |
| H2.6B | interpret the equations as lines and the common solution as the point of intersection |
| H2.7D | solve simultaneous equations in two unknowns, one equation being linear and the other being quadratic |

POSSIBLE SUCCESS CRITERIA

Solve the simultaneous equations $2x + 5y = -14$; $3x - 4y = 25$

Solve the simultaneous equations $x^2 + y^2 = 18$; $2x + 1 = y$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve a pair of simultaneous equations in a real-life context, such as 2 adult tickets and 1 child ticket cost £28, and 1 adult ticket and 3 child tickets cost £34. How much does 1 adult ticket cost?

Link the solution of simultaneous equations to their graphical representation.

COMMON MISCONCEPTIONS

Some students always discard solutions with negative values.

NOTES

Reinforce the fact that some problems may produce one inappropriate solution which can be ignored.

Clear presentation of working out is essential.

Link with graphical representations.

EXAMPLE QUESTIONS FROM SAMs: 4H q9

18. Function notation**Teaching time**

6-8 hours

OBJECTIVES

| | |
|--------------|--|
| H3.2A | understand the concept that a function is a mapping between elements of two sets |
| H3.2B | use function notations of the form $f(x) = \dots$ and $f : x \rightarrow \dots$ |
| H3.2C | understand the terms 'domain' and 'range' and which values may need to be excluded from a domain |
| H3.2D | understand and find the composite function fg and the inverse function f^{-1} |

POSSIBLE SUCCESS CRITERIA

Given $f(x) = 3 - 5x$; find $f(2)$, $f^{-1}(3)$

Given $g(x) = \frac{2}{3-x}$, write down the value of x that must be omitted from any domain of g .

Find $fg(4)$; $gf(4)$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming and solving equations using functions. E.g. solve $f(x) = g(x)$

Give the graph of $f(x)$ and use that to find $f(3)$ and $f(x) = 2$

COMMON MISCONCEPTIONS

Confusing $gf(x)$ with $fg(x)$

NOTES

Link with algebraic manipulation and equation solving.

EXAMPLE QUESTIONS FROM SAMs: 4H q17

19. Calculus**Teaching time**

7-9 hours

OBJECTIVES

| | |
|--------------|---|
| H3.4A | understand the concept of a variable rate of change |
| H3.4B | differentiate integer powers of x |
| H3.4C | determine gradients, rates of change, stationary points, turning points (maxima and minima) by differentiation and relate these to graphs |
| H3.4D | distinguish between maxima and minima by considering the general shape of the graph only |
| H3.4E | apply calculus to linear kinematics and to other simple practical problems |

POSSIBLE SUCCESS CRITERIA

Differentiate $8x^3 + 3x + 2$; $\frac{2}{x^2} + 3x$

Find the turning point of $y = x^2 + 8x - 20$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the values of x for which the graph of $y = x^2 - x + 3$ has a gradient of 7

Given that $s = t^3 + 2t^2$ find the value of t for which the particle is instantaneously at rest.

COMMON MISCONCEPTIONS

3 differentiates to 3 (rather than 0)

NOTES

Link with solving linear and quadratic equations.

EXAMPLE QUESTIONS FROM SAMs: 3H q21; 4H q25

20. Compound measures**Teaching time**

4-6 hours

OBJECTIVES

| | |
|---------------|--|
| F4.4G | use compound measure such as speed, density and pressure |
| F4.9A | convert measurements within the metric system to include linear and area units |
| F4.10A | convert between units of volume within the metric system |

POSSIBLE SUCCESS CRITERIA

Find the speed given distance and time.

Find the distance (in km) given the speed (in km/h) and the time (in minutes).

Recall and use the formula for density.

Give the formula for pressure, use it to find one of the variables.

Change 4 m^2 into cm^2 .

Change 45 mm^2 into cm^2 .

Change 3000 cm^3 into m^3 .

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Find the mass of an object, having first to find its volume.

Work out the average speed of a journey.

COMMON MISCONCEPTIONS

Using inconsistent units when solving problems.

Converting time into a decimal incorrectly. E.g. writing 1 hour 15 minutes as 1.15 hours.

NOTES

Practice converting time into decimals.

Ensure that conversions between metric units are known.

Ensure that consistent units are used when solving problems.

EXAMPLE QUESTIONS FROM SAMs: 3H q1; 4H q3

21. Geometry of shapes**Teaching time**

5-7 hours

OBJECTIVES

| | |
|--------------|--|
| F4.1B | use angle properties of intersecting lines, parallel lines and angles on a straight line |
| F4.1D | understand the terms 'isosceles', 'equilateral' and 'right-angled triangles' and the angle properties of these triangles |
| F4.2B | understand and use the term 'quadrilateral' and the angle sum property of quadrilaterals |
| F4.2C | understand and use the properties of the parallelogram, rectangle, square, rhombus, trapezium and kite |
| F4.2D | understand the term 'regular polygon' and calculate interior and exterior angles of regular polygons |
| F4.2E | understand and use the angle sum of polygons |
| H4.7A | provide reasons, using standard geometrical statements, to support numerical values for angles obtained in any geometrical context involving lines, polygons and circles |

POSSIBLE SUCCESS CRITERIA

Name all quadrilaterals that have a specific property.

Given the size of its exterior angle, how many sides does the polygon have?

What is the same, and what is different between families of polygons?

Given a geometric diagram, find the value of a given angle and give a reason for each stage of working.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step "angle chasing"-style problems that involve justifying how students have found a specific angle will provide opportunities to develop a chain of reasoning.

Geometrical problems involving algebra whereby equations can be formed and solved allow students the opportunity to make and use connections with different parts of mathematics.

COMMON MISCONCEPTIONS

Some students will think that all trapezia are isosceles, or a square is only square if 'horizontal', or a 'non-horizontal' square is called a diamond.

Incorrectly identifying the 'base angles' (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.

NOTES

Students must be encouraged to use geometrical language appropriately, 'quote' the appropriate reasons for angle calculations and show step-by-step deduction when solving multi-step problems.

Emphasise that diagrams in examinations are seldom drawn accurately.

Use triangles to find angle sums of polygons; this could be explored algebraically as an investigation.

EXAMPLE QUESTIONS FROM SAMs: 4H q11

22. Constructions and bearings**Teaching time**

3-5 hours

OBJECTIVES

| | |
|---------------|--|
| F4.5B | construct triangles and other two-dimensional shapes using a combination of a ruler, a protractor and compasses |
| F4.5D | use straight edge and compasses to: (i) construct the perpendicular bisector of a line segment (ii) construct the bisector of an angle |
| F4.4D | understand angle measure including three-figure bearings |
| F4.5C | solve problems using scale drawings |
| F4.11B | use and interpret maps and scale drawings |

POSSIBLE SUCCESS CRITERIA

Able to read and construct scale drawings.

When given the bearing of a point A from point B , can work out the bearing of B from A .

Know that scale diagrams, including bearings and maps, are 'similar' to the real-life examples.

Construct the perpendicular bisector of a given angle.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving combinations of bearings and scale drawing can provide a rich opportunity to link with other areas of mathematics and allow students to justify their findings.

COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

NOTES

Drawings should be done in pencil.

Construction lines should not be erased.

EXAMPLE QUESTIONS FROM SAMs: 3H q5

23. Perimeter, area and volume**Teaching time**

7-9 hours

OBJECTIVES

| | |
|---------------|--|
| F4.9B | find the perimeter of shapes made from triangles and rectangles |
| F4.9C | find the area of simple shapes using the formulae for the areas of triangles and rectangles |
| F4.9D | find the area of parallelograms and trapezia |
| H4.9A | find perimeters and areas of sectors of circles |
| F4.10C | find the surface area of simple shapes using the area formulae for triangles and rectangles |
| F4.10D | find the surface area of a cylinder |
| F4.10E | find the volume of prisms, including cuboids and cylinders, using an appropriate formula |
| H4.10A | find the surface area and volume of a sphere and a right circular cone using relevant formulae |

POSSIBLE SUCCESS CRITERIA

Calculate the area and/or perimeter of shapes with different units of measurement.

Understand that answers in terms of π are more accurate.

Calculate the perimeters and/or areas of circles and sectors of circles given the radius or diameter and vice versa.

Work out the length given the area of the cross-section and volume of a cuboid.

Given two solids with the same volume and the dimensions of one, write and solve an equation in terms of π to find the dimensions of the other.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using compound shapes or combinations of polygons that require students to subsequently interpret their result in a real-life context.

Multi-step problems, including the requirement to form and solve equations, provide links with other areas of mathematics.

Combinations of 3D forms such as a cone and a sphere where the radius has to be calculated given the total height.

COMMON MISCONCEPTIONS

Students often get the concepts of area and perimeter confused.

Students often get the concepts of surface area and volume confused.

NOTES

Encourage students to draw a sketch where one isn't provided.

Ensure that examples use different metric units of length, including decimals.

Emphasise the need to learn the circle formulae; "Cherry Pie's Delicious" and "Apple Pies are too" are good ways to remember them.

Ensure that students know it is more accurate to leave answers in terms of π , but only when asked to do so.

EXAMPLE QUESTIONS FROM SAMs: 3H q10, 19; 4H q3

24. Pythagoras' Theorem and trigonometry**Teaching time**

7-9 hours

OBJECTIVES

| | |
|--------------|--|
| F4.8A | know, understand and use Pythagoras' Theorem in two dimensions |
| F4.8B | know, understand and use sine, cosine and tangent of acute angles to determine lengths and angles of a right-angled triangle |
| F4.8C | apply trigonometrical methods to solve problems in two dimensions |
| H4.8A | understand and use sine, cosine and tangent of obtuse angles |
| H4.8B | understand and use angles of elevation and depression |

POSSIBLE SUCCESS CRITERIA

Does 2, 3, 6 give a right-angled triangle?

Justify when to use Pythagoras' Theorem and when to use trigonometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' Theorem or a combination of Pythagoras' Theorem and the trigonometric ratios.

Link to 'real-life' situations E.g. link with bearings and scale drawings.

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.

Students forget to square root their final answer, or round their answer prematurely.

NOTES

Students may need reminding about surds.

Scale drawings are not acceptable.

Calculators need to be in degree mode.

Use a suitable mnemonic to remember SOHCAHTOA.

Use Pythagoras' Theorem and trigonometry together.

EXAMPLE QUESTIONS FROM SAMs: 3H q7

25. Transformations**Teaching time**

4-6 hours

OBJECTIVES

| | |
|--------------|--|
| F5.2A | understand that rotations are specified by a centre and an angle |
| F5.2B | rotate a shape about a point through a given angle |
| F5.2C | recognize that an anti-clockwise rotation is a <i>positive</i> angle of rotation and a clockwise rotation is a <i>negative</i> angle of rotation |
| F5.2D | understand that reflections are specified by a mirror line |
| F5.2E | construct a mirror line given an object and reflect a shape given a mirror line |
| F5.2F | understand that translations are specified by a distance and direction |
| F5.2G | translate a shape |
| F5.2H | understand and use column vectors in translations |
| F5.2I | understand that rotations, reflections and translations preserve length and angle so that a transformed shape under any of these transformations remains congruent to the original shape |
| F5.2J | understand that enlargements are specified by a centre and a scale factor |
| F5.2K | understand that enlargements preserve angles and not lengths |
| F5.2L | enlarge a shape given the scale factor |
| F5.2M | identify and give complete descriptions of transformations |

POSSIBLE SUCCESS CRITERIA

Understand that translations are specified by a distance and direction (using a vector).

Recognise that enlargements preserve angle but not length.

Understand that distances and angles are preserved under rotations, reflections and translations so that any shape is congruent to its image.

Understand that similar shapes are enlargements of each other and angles are preserved.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

COMMON MISCONCEPTIONS

Students often use the term 'transformation' when describing transformations instead of the required information.

Lines parallel to the coordinate axes often get confused.

NOTES

Emphasise the need to describe the transformations fully, and if asked to describe a 'single' transformation students should not include two types.

Find the centre of rotation, by trial and error and by using tracing paper. Include centres on or inside shapes.

EXAMPLE QUESTIONS FROM SAMs: 4H q6

26. Circle theorems**Teaching time**

5-7 hours

OBJECTIVES

| | |
|--------------|---|
| H4.6A | understand and use the internal and external intersecting chord properties |
| H4.6B | recognise the term 'cyclic quadrilateral' |
| H4.6C | understand and use angle properties of the circle including: <ul style="list-style-type: none"> (i) angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the remaining part of the circumference (ii) angle subtended at the circumference by a diameter is a right angle (iii) angles in the same segment are equal (iv) the sum of the opposite angles of a cyclic quadrilateral is 180° (v) the alternate segment theorem |

POSSIBLE SUCCESS CRITERIA

Justify clearly missing angles on diagrams using the various circle theorems, giving a reason for each stage in working.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that involve a clear chain of reasoning and provide counter-arguments to statements.

Can be linked to other areas of mathematics by incorporating trigonometry and Pythagoras' Theorem.

COMMON MISCONCEPTIONS

Much of the confusion arises from mixing up the diameter and the radius.
There is often confusion when identifying cyclic quadrilaterals.

NOTES

Reasoning needs to be carefully constructed and correct notation should be used throughout. Students should label any diagrams clearly, as this will assist them; particular emphasis should be made on labelling any radii in the first instance.

EXAMPLE QUESTIONS FROM SAMs: 3H q16

27. Advanced trigonometry**Teaching time**

7-9 hours

OBJECTIVES

| | |
|--------------|---|
| H4.8C | understand and use the sine and cosine rules for any triangle |
| H4.8D | use Pythagoras' theorem in three dimensions |
| H4.8E | understand and use the formula $\frac{1}{2}ab \sin C$ for the area of a triangle |
| H4.8F | apply trigonometrical methods to solve problems in three dimensions, including finding the angle between a line and a plane |

POSSIBLE SUCCESS CRITERIA

Find the area of a segment of a circle given the radius and length of the chord.

Justify when to use the cosine rule, sine rule, Pythagoras' Theorem or normal trigonometric ratios to solve problems.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Triangles formed in a semi-circle can provide links with other areas of mathematics.

Multi-step problems requiring the use of both the Sine rule and Cosine rule.

COMMON MISCONCEPTIONS

Not using the correct rule, or attempting to use 'normal trig' in non-right-angled triangles.

When finding angles students will often be unable to rearrange the cosine rule or fail to find the inverse of $\cos \theta$.

NOTES

The cosine rule is used when we have SAS and used to find the side opposite the 'included' angle or when we have SSS to find an angle.

Ensure that finding angles with 'normal trig' is refreshed prior to this topic.

Students may find it useful to be reminded of simple geometrical facts, i.e. the shortest side is always opposite the shortest angle in a triangle.

In multi-step questions emphasise the importance of not rounding prematurely and using exact values where appropriate.

EXAMPLE QUESTIONS FROM SAMs: 3H q19; 4H q21

28. Similar shapes**Teaching time**

6-8 hours

OBJECTIVES

| | |
|---------------|---|
| F4.2F | understand congruence as meaning the same shape and size |
| F4.2G | understand that two or more polygons with the same shape and size are said to be congruent to each other |
| F4.11A | understand and use the geometrical properties that similar figures have corresponding lengths in the same ratio but corresponding angles remain unchanged |
| H4.11A | understand that areas of similar figures are in the ratio of the square of corresponding sides |
| H4.11B | understand that volumes of similar figures are in the ratio of the cube of corresponding sides |
| H4.11C | use areas and volumes of similar figures in solving problems |

POSSIBLE SUCCESS CRITERIA

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not.

Understand that enlargement does not have the same effect on area and volume.

Given the volumes of two similar shapes and the surface area of one, find the surface area of the other shape.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step questions which require calculating missing lengths of similar shapes prior to calculating area of the shape, or using this information in trigonometry or Pythagoras problems.

COMMON MISCONCEPTIONS

Students commonly use the same scale factor for length, area and volume.

NOTES

Encourage students to model consider what happens to the area when a 1 cm square is enlarged by a scale factor of 3.

Ensure that examples involving given volumes are used, requiring the cube root being calculated to find the length scale factor.

EXAMPLE QUESTIONS FROM SAMs: 4H q18

29. Vectors**Teaching time**

5-7 hours

OBJECTIVES

| | |
|--------------|---|
| H5.1A | understand that a vector has both magnitude and direction |
| H5.1B | understand and use vector notation including column vectors |
| H5.1C | multiply vectors by scalar quantities |
| H5.1D | add and subtract vectors |
| H5.1E | calculate the modulus (magnitude) of a vector |
| H5.1F | find the resultant of two or more vectors |
| H5.1G | apply vector methods for simple geometrical proofs |

POSSIBLE SUCCESS CRITERIA

Add and subtract vectors algebraically and use column vectors.

Solve geometric problems and produce proofs.

Find the magnitude of a vector.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

"Show that"-type questions are an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics, in particular algebra.

Find the area of a parallelogram defined by given vectors.

COMMON MISCONCEPTIONS

Students find it difficult to understand that parallel vectors are equal as they are in different locations in the plane.

NOTES

Students find manipulation of column vectors relatively easy compared to pictorial and algebraic manipulation methods – encourage them to draw any vectors they calculate on the picture.

Geometry of a hexagon provides a good source of parallel, reverse and multiples of vectors.

Remind students to underline vectors or use an arrow above them, or they will be regarded as just lengths.

Extend geometric proofs by showing that the medians of a triangle intersect at a single point.

EXAMPLE QUESTIONS FROM SAMs: 3H q22; 4H q23

30. Graphical representation of data**Teaching time**

4-6 hours

OBJECTIVES

| | |
|--------------|---|
| H6.1A | construct and interpret histograms |
| H6.1B | construct cumulative frequency diagrams from tabulated data |
| H6.1C | use cumulative frequency diagrams |

POSSIBLE SUCCESS CRITERIA

Construct cumulative frequency graphs and histograms from frequency tables.

Compare two data sets and justify their comparisons based on measures extracted from their diagrams where appropriate in terms of the context of the data.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpret two or more data sets from cumulative frequency graphs and relate the key measures in the context of the data.

COMMON MISCONCEPTIONS

Labelling axes incorrectly in terms of the scales, and also using 'Frequency' instead of 'Frequency Density' or 'Cumulative Frequency'.

Students often confuse the methods involved with cumulative frequency, estimating the mean and histograms when dealing with data tables.

Histograms are often not well understood with the height used for frequency rather than the area.

NOTES

Ensure that axes are clearly labelled.

EXAMPLE QUESTIONS FROM SAMs: 3H q14; 4H q12

31. Statistical measures**Teaching time**

3-5 hours

OBJECTIVES

| | |
|--------------|--|
| F6.2A | understand the concept of average |
| F6.2B | calculate the mean, median, mode and range for a discrete data set |
| F6.2C | calculate an estimate for the mean for grouped data |
| F6.2D | identify the modal class for grouped data |
| H6.2A | estimate the median from a cumulative frequency diagram |
| H6.2B | understand the concept of a measure of spread |
| H6.2C | find the interquartile range from a discrete data set |
| H6.2D | estimate the interquartile range from a cumulative frequency diagram |

POSSIBLE SUCCESS CRITERIA

Be able to state the median, mode, mean and range from a small data set.

Be able to find the interquartile range from a discrete data set.

Estimate the mean from a grouped frequency table.

Estimate the median and interquartile range from a cumulative frequency graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide reasons for choosing to use a specific average to support a point of view.

Given the mean, median and mode of five positive whole numbers, can you find the numbers?

Students should be able to provide a correct solution as a counter-argument to statements involving the “averages”, e.g. Susan states that the median is 15, she is wrong. Explain why.

Find the median from a histogram.

COMMON MISCONCEPTIONS

Students often forget the difference between continuous and discrete data.

Often the $\sum(m \times f)$ is divided by the number of classes rather than $\sum f$ when estimating the mean.

NOTES

Encourage students to cross out the midpoints of each group once they have used these numbers to in $m \times f$. This helps students to avoid summing m instead of f .

Remind students how to find the midpoint of two numbers.

Emphasise that continuous data is measured, i.e. length, weight, and discrete data can be counted, i.e. number of shoes.

EXAMPLE QUESTIONS FROM SAMs: 3H q3ab, q12; 4H q7, q12

OBJECTIVES

| | |
|--------------|---|
| F6.3C | understand and use estimates or measures of probability from theoretical models |
| F6.3D | find probabilities from a Venn diagram |
| F6.3E | understand the concepts of a sample space and an event, and how the probability of an event happening can be determined from the sample space |
| F6.3G | estimate probabilities from previously collected data |
| F6.3H | calculate the probability of the complement of an event happening |
| F6.3I | use the addition rule of probability for mutually exclusive events |
| F6.3J | understand and use the term 'expected frequency' |
| H6.3A | draw and use tree diagrams |

POSSIBLE SUCCESS CRITERIA

If the probability of outcomes are x , $2x$, $4x$, $3x$, calculate x .

Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French given that they also study German.

Use a tree diagram to find the probability of a combined event.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to justify the probability of events happening or not happening in real-life and abstract contexts.

COMMON MISCONCEPTIONS

Probability without replacement is best illustrated visually and by initially working out probability 'with' replacement.

Not using fractions or decimals when working with probability trees.

NOTES

Encourage students to work 'across' the branches, working out the probability of each successive event. The probability of the combinations of outcomes should = 1.

If a question says, for example, that 'two counters are taken from a bag' then, by implication, this is a non-replacement probability question.

EXAMPLE QUESTIONS FROM SAMs: 3H q3c; 4H q15bc, q20

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